

Midterm I Practice Problems

Midterm II will cover: Section 4.4, 4.5, 4.6, 4.7, 7.2, 7.3, 7.4, 7.5, 7.6. The following topics are particularly important:

Solve Caychy-Euler equation;

Use the method of undetermined coefficients to solve nonhomogeneous equations;

Use the method of variation of parameters to solve nonhomogeneous equations;

Compute Laplace transform;

Compute the inverse Laplace transform;

Solve differential equation using Laplace transform;

Laplace transform of discontinuous functions.

1. Solve the initial value problem $x^2y'' - 2xy' + 2y = 0$, $y(1) = 2$, $y'(1) = 3$.
2. Use the method of undetermined coefficients to solve the following nonhomogeneous equation. Find the general solution.

$$y'' - 2y' - 3y = 2e^{3t}.$$

3. Use variation of parameters to solve the following nonhomogeneous equation. Find the general solution.

$$y'' + 4y = \sec(2t)$$

4. Determine

$$\mathcal{L}^{-1}\left\{\frac{3s^2 + 5s + 3}{s^4 + s^3}\right\}.$$

5. Express the given function using window and step functions and compute its Laplace transform.

$$f(t) = \begin{cases} 0, & t \in [0, \pi/2), \\ \sin t, & t \in [\pi/2, \infty). \end{cases}$$

6. Solve the given initial value problem using the method of Laplace transforms. Sketch the graph of the solution.

$$y'' + y = u(t - 2) - u(t - 4), \quad y(0) = 1, \quad y'(0) = 0.$$