

Spring 2021 Math 20D Lecture B Homework #1

Due Friday 11:59pm, April 9th

Submit this homework through Gradescope, check the course website on how to submit homework.

Topics covered: section 1.2, 2.2-2.4

1. Verify that the given relation is an implicit solution to the given differential equation.

$$y - \ln y = x^2 + 1, \quad \frac{dy}{dx} = \frac{2xy}{y-1}.$$

2. Verify that the function $\phi(x) = c_1e^x + c_2e^{-2x}$ is a solution to the given equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$$

for any choice of the constants c_1 and c_2 . Determine c_1 and c_2 so that the following initial conditions is satisfied:

$$y(1) = 1, \quad y'(1) = 0.$$

3. Solve the initial value problem

$$\sqrt{y}dx + (1+x)dy = 0, \quad y(0) = 1.$$

4. Solve the initial value problem

$$y' = x^3(1+y), \quad y(0) = 3.$$

5. According to Newton's law of cooling, if an object at temperature T is immersed in a medium having the constant temperature M , then the rate of change of T is proportional to the difference of temperature $M - T$. This gives the differential equation

$$\frac{dT}{dt} = k(M - T).$$

- (a) Solve the differential equation for T .
 - (b) A thermometer reading 100°F is placed in a medium having a constant temperature of 70°F . After 6 min, the thermometer reads 80°F . What is the reading after 20 min? You can leave your answer unsimplified.
6. Solve the initial value problem

$$\frac{dy}{dx} + \frac{3y}{x} + 2 = 3x, \quad y(1) = 1.$$

7. Solve the initial value problem

$$(\sin x) \frac{dy}{dx} + y \cos x = x \sin x, \quad y(\pi/2) = 2.$$

8. Solve the initial value problem

$$(ye^{xy} - 1/y)dx + (xe^{xy} + x/y^2)dy = 0, \quad y(1) = 1.$$

9. Consider the differential equation and initial condition

$$\alpha tx + (3t^2 + 2\cos(x)\sin(x))x' = 0, \quad x(1) = \pi/4.$$

- (a) Find a value of the parameter α such that the equation is exact.
- (b) For this value of α , find the solution of the initial value problem in implicit form.

Note: Due to the number of students in this class, a subset of the questions will be used to grade this assignment. This subset will be determined after all of the assignments have been submitted.