

**Spring 2021 Math 20D Lecture B Homework #4**  
Due Sunday, 11:59pm, May 2nd

Submit this homework through Gradescope.

**Topics covered: section 4.6, 4.7, 7.2**

1. Find the general solution to the differential equation

$$y'' - 6y' + 9y = t^{-3}e^{3t}.$$

2. Find the general solution to the differential equation for  $x > 0$ .

$$\frac{d^2y}{dx^2} + \frac{6}{x} \frac{dy}{dx} + \frac{4}{x^2}y = 0.$$

3. Solve the initial value problem for the Cauchy-Euler equation.

$$x^2y'' - 3xy' + 5y = 0, \quad y(1) = -1, \quad y'(1) = 4.$$

4. Devise a modification of the method for Cauchy-Euler equation to find two linearly independent solutions to the given equation.

$$(x + 1)^2y'' + 10(x + 1)y' + 14y = 0, \quad x > -1.$$

(Hint: Try  $y_p = (x + 1)^r$ ).

5. Find the general solution to the non-homogeneous Cauchy-Euler equation using variation of parameters.

$$x^2y'' + 3xy' + y = \frac{1}{x}.$$

6. Let

$$f(t) = \begin{cases} 1 - t, & 0 \leq t < 1, \\ 0, & t \geq 1. \end{cases}$$

Use the definition of Laplace transform to find  $\mathcal{L}\{f(t)\}$ .

**Note:** Due to the number of students in this class, a subset of the questions will be used to grade this assignment. This subset will be determined after all of the assignments have been submitted.