## Spring 2021 Math 20D Lecture B Homework \#4

Due Sunday, 11:59pm, May 9th

Submit this homework through Gradescope.
Topics covered: section $7.3,7.4,7.5$

1. Determine $\mathscr{L}\left\{e^{7 t} \cos ^{2} t\right\}$ and $\mathscr{L}\left\{t e^{7 t} \cos ^{2} t\right\}$. You can leave your answer unsimplified. Hint: $\cos ^{2} t=\frac{1}{2} \cos (2 t)+\frac{1}{2}$.
2. Determine

$$
\mathscr{L}^{-1}\left\{\frac{3}{(2 s+5)^{3}}\right\} .
$$

3. Determine

$$
\mathscr{L}^{-1}\left\{\frac{7 s^{2}-41 s+84}{(s-1)\left(s^{2}-4 s+13\right)}\right\} .
$$

4. Solve the given initial value problem using the method of Laplace transforms.

$$
y^{\prime \prime}+6 y^{\prime}+5 y=12 e^{t}, \quad y(0)=-1, y^{\prime}(0)=7 .
$$

5. Solve the given initial value problem using the method of Laplace transforms.

$$
t y^{\prime \prime}-2 y^{\prime}+t y=0 ; \quad y(0)=1, y^{\prime}(0)=0 .
$$

Hint: $\mathscr{L}^{-1}\left\{1 /\left(s^{2}+1\right)^{2}\right\}=(\sin t-t \cos t) / 2$. No worries if you cannot get rid of (or determine the value of) the constant $C$ in your final answer. The solution to this initial value problem can be non-unique.

