HW8 Problems

1. Let $f = \frac{1}{(z-1)(z-5)}$.

(a). Prove that there is a sequence of rational functions $R_n(z)$ whose poles can only occur at 2 and 6 such that

$$\lim_{n\to\infty} \sup_{3\le|z|\le 4} |f(z) - R_n(z)| = 0. \tag{1}$$

(b). Does there exist a sequence of rational functions $R_n(z)$ whose poles can only occur at 6 such that (1) holds? Justify your answer.

2. Let $G = \{z \in \mathbb{C} : |z| < 1 \text{ and } |z - \frac{1}{3}| > \frac{2}{3}\}$; and K be the closure of G : $K = \{z \in \mathbb{C} : |z| \le 1 \text{ and } |z - \frac{1}{3}| \ge \frac{2}{3}\}$. Let $A(K)$ be the space of continuous functions on K that are analytic on G equipped with the uniform norm on K. For the purposes of this problem, a Laurent polynomial is a function of the form $\sum_{n=-N}^{N} a_n z^n$. Determine whether the following are true or false. Justify your answer.

(a). The set of polynomials is dense in $H(G)$.

(b). The set of polynomials is dense in $A(K)$.

(c). If f is analytic on a neighborhood of K, then f can be uniformly approximated on K by Laurent polynomials.