Math 180A - Homework 1
(Due Thursday, Jan 18, 5:00 PM)

Reading: Sections 2.1 - 2.4 of the textbook and lecture notes 1-3.

Show FULL JUSTIFICATION for all your answers.

Warm-Up Exercises (Do not turn in)

1. Theoretical Exercises 1 - 4 of Chapter 2 of the textbook.

2. Theoretical Exercise 6. (a), (b), (d), (e), (g), (i), (j) of Chapter 2. of the textbook. It may help to draw a Venn diagrams for each part.

Homework Problems (To turn in)

1. Two coins are flipped and a die is rolled.
   (a) Write the sample space for this experiment.
   (b) Let $A$ be the event that both coins will be heads. Write the outcomes in $A$.
   (c) Let $B$ be the event that result of the dice roll will be an odd number. Write the outcomes in $B$.

2. Given events $A$ and $B$,
   (a) let $C$ be the event that $A$ will occur and $B$ will not occur. Express $C$ in terms of $A$ and $B$. Let $D$ be the event that $B$ will occur and $A$ will not occur. Express $D$ in terms of $A$ and $B$.
   (b) let $E$ be the event that exactly one of the events $A$ or $B$ will occur. Express $E$ in terms of $A$ and $B$.
   (c) Use the result in (a) to find a formula for the probability of $E$ in terms of the quantities $P(A)$, $P(B)$ and $P(A \cap B)$.
   (d) Compare the resulting formula with the formula for $P(F)$, where $F$ is the event that at least one of the events $A$ or $B$ will occur.

3. Theoretical Exercise 6. (c), (f), (h) of Chapter 2. of the textbook. It may help to draw a Venn diagrams for each part.

4. Alice and Bob plan to meet, and each of them will arrive at a random time between 1:00pm and 2:00pm.
   (a) Write the sample space for the pair of arrival times of Alice and Bob.
   (b) Write the event $A$ in which they both arrive before 1:30pm.
   (c) Write the event $B$ in which Alice arrives before Bob.

5. Keith buys a lottery ticket in three different lotteries. Let $A_i$ be the event that Keith wins the $i$th lottery ($i = 1, 2, 3$). Express the following events in terms of the events $A_i$; $i = 1, 2, 3$: 
(a) Keith wins exactly two lotteries
(b) Keith wins at least one lottery
(c) Keith wins no more than two lotteries

6. **Problem** 9. of Chapter 2. of the textbook.

7. **Problem** 11. of Chapter 2. of the textbook.

8. **Theoretical Exercise** 11. of Chapter 2. of the textbook.

9. Consider the sample space \( \Omega = \{1, 2, 3, 4, \ldots \} \) (the set of all natural numbers). We want to show that there is no probability measure on \( \Omega \) under which “all outcomes are equally likely”. Let’s argue by contradiction. Suppose \( P \) is a probability measure such that \( P(\{n\}) \) has the same value for all \( n \in \Omega \). Let’s see what can go wrong.

   (a) Suppose \( P(\{n\}) > 0 \). Which axiom of probability will be violated?
   (b) Suppose \( P(\{n\}) = 0 \). Which axiom of probability will be violated?

**More Practice (Do not turn in)**

1. **Problem** 2. of Chapter 2. of the textbook.

2. **Problem** 4. of Chapter 2. of the textbook.

3. **Problem** 12. of Chapter 2. of the textbook.