Review of Probability Distributions

In this course, we have introduced six different probability distributions, three for discrete random variables and three for continuous random variables. Below is a brief review of these distributions.

- A random variable $X$ has the geometric distribution with parameter $p$ if
  \[ P(X = k) = (1 - p)^{k-1}p, \quad k = 1, 2, \ldots. \]
  This is the distribution of the number of trials required to get a success if each trial is independently successful with probability $p$.

- A random variable $X$ has the binomial distribution with parameters $n$ and $p$ if
  \[ P(X = k) = \binom{n}{k}p^k(1-p)^{n-k}, \quad k = 0, 1, \ldots, n. \]
  This is the distribution of the number of successes in $n$ trials if each trial is independently successful with probability $p$.

- A random variable $X$ has the Poisson distribution with parameter $\lambda$ if
  \[ P(X = k) = e^{-\lambda}\frac{\lambda^k}{k!}, \quad k = 0, 1, 2, \ldots. \]
  This is the distribution of the number of events that should occur during a time interval, if we expect $\lambda$ occurrences on average and if events occur at a constant rate.

- A random variable $X$ has the uniform distribution on $[a, b]$ if it has density
  \[ f(x) = \begin{cases} 1/(b - a) & \text{if } a \leq x \leq b \\ 0 & \text{otherwise} \end{cases} \]
  Informally, this means that $X$ is equally likely to be anywhere between $a$ and $b$.

- A random variable $X$ has the exponential distribution with parameter $\lambda$ if it has density
  \[ f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases} \]
  This is the distribution of the time to wait for the next event to occur, if events occur at the constant rate $\lambda$ per unit time.

- A random variable $X$ has the normal distribution with mean $\mu$ and standard deviation $\sigma$ if it has density
  \[ f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty. \]
  This is a symmetric, bell-shaped distribution which arises frequently in practice because of the Central Limit Theorem.