Math 20C **Name:** ______________________________
Summer II 2016
Final Exam Solutions
09/03/2016
Time Limit: 3 Hours

This exam contains 10 questions. Be sure to write your name, PID, and section on the front of your blue book. This is a closed note, closed book exam. Calculators are not allowed. Show all of your work. No credit will be given for unsupported answers, even if correct. Simplify answers as much as possible.

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1. (10 points) Marty Math realizes he's late to his final exam for 20C. He runs like crazy toward the lecture hall according to the path $c(t) = (t, 2t, \ln(t/5))$. Suppose that his stress level at position (x, y, z) is given by $x^2y + 5z$. If Marty makes it to the lecture hall when $t = 5$, what is the rate of change of his stress level with respect to time when he arrives for the exam?

   Solution: 151

2. (10 points) Estimate the value of $2.02e^{.01}$ using a linear approximation of a function of two variables.

   Solution: 2.04

3. (10 points) Find an equation for the plane tangent to $xyz = 6$ at the point $(1, 2, 3)$.

   Solution: $6x + 3y + 2z = 18$

4. (10 points) Parametrize the line of intersection of the planes $x+3y+z = 0$ and $y-x = 1$.

   Solution: $\ell(t) = (0, 1, -3) + t(-1, -1, 4)$

5. (10 points) The acceleration of a particle is given by $a(t) = (1, \cos(t), 6t)$. Suppose that the initial velocity of the particle is $v(0) = (-1, 2, 3)$ and the initial position of the particle is $c(0) = (0, 0, 0)$.

   (a) Give an equation for $v(t)$, the velocity of the particle at time t.
   Solution: $v(t) = (t - 1, \sin(t) + 2, 3t^2 + 3)$

(b) Give an equation for $c(t)$, the position of the particle at time t.
Solution: $c(t) = \left(\frac{1}{2}t^2 - t, -\cos(t) + 2t + 1, t^3 + 3t\right)$

(c) At what times does the particle intersect the yz plane?
Solution: $t = 0, 2$

6. (10 points) Compute the following limit or say why it doesn't exist:

$$\lim_{(x,y)\to(0,0)} \frac{x^6 - y^6}{x^3 + y^3}.$$

Solution: Factor the numerator as $(x^3+y^3)(x^3-y^3)$. Things cancel, and you're left with $x^3 - y^3$ which is continuous, so the limit is 0.

7. (10 points) Using the Lagrange multipliers method, find all points where extrema of $xy^2 + 2x$ may occur, subject to the constraint $x^2 + xy^2 = 4$.

Solution: The critical points are $(-2, 0), (2, 0), (1, \sqrt{3}), (1, -\sqrt{3})$.

8. (10 points) Suppose that as you move away from the point $(2, 0, 1)$, the function $f(x, y, z)$ decreases most rapidly in the direction $(2, 2, 1)$. If the rate of change of f in this direction is -6 per unit distance, what is $\nabla f(2, 0, 1)$?

Solution: $(-4, -4, -2)$

9. (10 points) Compute the following integral:

$$\int_0^4 \int_{\sqrt{y}}^2 3\cos(x^3 + 1)\, dxdy.$$

Note: It's okay if your answer involves trig functions.

Solution: $\sin(9) - \sin(1)$

10. (10 points) A cylinder of radius r is defined by the equation $x^2 + y^2 = r^2$. Compute the volume of the cylindrical wedge consisting of points in the cylider such that $0 \le z \le y$.

Solution:

$$\int_{-r}^{r} \int_0^{\sqrt{r^2-x^2}} \int_0^y 1 dzdydx = \frac{2}{3}r^3.$$