

Math 20C
Summer II 2016
Exam 1
8/12/2016
Time Limit: 60 Minutes

Name: _____

This exam contains 5 questions. Be sure to write your name, PID, and section on the front of your blue book. This is a closed note, closed book exam. Calculators are not allowed. Show all of your work. No credit will be given for unsupported answers, even if correct. Simplify answers as much as possible.

1. (10 points) Compute an equation of the plane containing the points $(1, 2, 3)$ and $(4, 3, 2)$ and $(-1, 3, 1)$.

Solution: We first compute two vectors which lie in the plane: $(4, 3, 2) - (1, 2, 3) = (3, 1, -1)$ and $(-1, 3, 1) - (1, 2, 3) = (-2, 1, -2)$. Now we compute their cross product to obtain a normal vector to the plane:

$$(3, 1, -1) \times (-2, 1, -2) = (-1, 8, 5).$$

Thus, an equation of the plane is

$$-(x - 1) + 8(y - 2) + 5(z - 3) = 0.$$

2. (10 points) Let \mathbf{u} and \mathbf{v} be vectors such that $\|\mathbf{u}\| = 6$ and $\|\mathbf{v}\| = 2$. If $\mathbf{u} \cdot \mathbf{v} = -12$, what is $\|\mathbf{u} \times \mathbf{v}\|$?

Solution: We first use the fact that $\mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\|\|\mathbf{v}\|\cos(\theta)$. Plugging in the given values, we see that $-12 = 2(6)\cos(\theta)$ so $\cos(\theta) = -1$. This implies $\theta = \pi$. Now we use the fact that $\|\mathbf{u} \times \mathbf{v}\| = \|\mathbf{u}\|\|\mathbf{v}\|\sin(\theta)$. This gives:

$$\|\mathbf{u} \times \mathbf{v}\| = 2(6)\sin(\pi) = 0.$$

3. (10 points) For each of the following questions, write TRUE, FALSE, or NOT SURE. For all questions, \mathbf{u} , \mathbf{v} , and \mathbf{w} are vectors in \mathbb{R}^3 . You do not need to justify your answers. You will earn:

- 2 points for a correct answer
- 1 point for any question marked NOT SURE
- 0 points for an incorrect answer

- (a) If $\mathbf{v} \perp \mathbf{w}$, then $\mathbf{v} \times \mathbf{w} = (0, 0, 0)$.

False. A counterexample is $\mathbf{v} = (1, 0, 0)$ and $\mathbf{w} = (0, 1, 0)$.

(b) If $\mathbf{u} \perp \mathbf{v}$ and $\mathbf{v} \perp \mathbf{w}$ then $\mathbf{u} \perp \mathbf{w}$.

False. A counterexample is $\mathbf{u} = (1, 0, 0)$, $\mathbf{v} = (0, 1, 0)$ and $\mathbf{w} = (1, 0, 0)$.

(c) The path $c(t) = (-\sin(t), \cos(-t))$ traces out the unit circle in the counterclockwise direction as t varies from 0 to 2π .

True. Plugging in a few values shows you in which direction the circle is traced out.

(d) There is exactly one plane containing the points $(-1, 2, 4)$, $(3, -6, -12)$, and $(-2, 4, 8)$.

False. These points all lie on the same line since they are all scalar multiples of each other, so there are infinitely planes containing them.

(e) If \mathbf{u} and \mathbf{v} are orthogonal, then $\mathbf{u} \cdot \mathbf{v} = (0, 0, 0)$.

False. If \mathbf{u} and \mathbf{v} are orthogonal then $\mathbf{u} \cdot \mathbf{v} = 0$, not the zero vector.

4. (10 points) Estimate the value of $\cos(\pi(.51)(2.02)) + \ln(.51(2.02))$ using a linear approximation of a suitable function.

Solution: Let $f(x, y) = \cos(\pi xy) + \ln(xy)$. We'll start by finding a linear approximation $L(x, y)$ of f at the point $(1/2, 2)$, then use this nice function to compute $L(.51, 2.02)$.

$$\begin{aligned}f(x, y) &= \cos(\pi xy) + \ln(xy) \\ \frac{\partial f}{\partial x} &= -\pi y \sin(\pi xy) + \frac{1}{x} \\ \frac{\partial f}{\partial y} &= -\pi x \sin(\pi xy) + \frac{1}{y}\end{aligned}$$

Now we plug in $(x, y) = (1/2, 2)$:

$$\begin{aligned}f(1/2, 2) &= -1 \\ \frac{\partial f}{\partial x}(1/2, 2) &= 2 \\ \frac{\partial f}{\partial y}(1/2, 2) &= 1/2\end{aligned}$$

Putting this all together, the linear approximation is

$$L(x, y) = -1 + 2(x - 1/2) + \frac{1}{2}(y - 2)$$

so we have

$$f(.51, 2.02) \approx L(.51, 2.02) = -.97.$$

5. (10 points) Evaluate

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy^3 - x^3y}{x^4 + y^4}$$

or explain why it doesn't exist.

Solution: The limit does not exist. To see this, we'll show that the limiting values disagree along two lines of approach. Along the line $y = x$ the limiting value is:

$$\lim_{x \rightarrow 0} \frac{x^4 - x^4}{2x^4} = \lim_{x \rightarrow 0} \frac{0}{2x^4} = 0.$$

Along the line $y = 2x$ the limiting value is:

$$\lim_{x \rightarrow 0} \frac{x(2x)^3 - x^3(2x)}{x^4 + (2x)^4} = \lim_{x \rightarrow 0} \frac{6x^4}{17x^4} = \frac{6}{17}.$$