A Brief Review

The following are just a few of the basics you are expected to know. If you’ve forgotten or are confused by any of this, please review on your own or ask for help immediately. In the rest of this document, the letter $C$ will always denote a constant.

Graphing Functions. You should be familiar with the graphs of the following functions. Know where they are defined, $x$- and $y$- intercepts, basic shape, and asymptotic behavior.

- $C$
- $x$
- $x^2$
- $\sqrt{x}$
- $|x|$
- $1/x$
- $e^x$
- $\ln x$
- $\sin(x)$
- $\cos(x)$
- $\tan(x)$

Logarithm and Exponentiation Facts.

- $\log_b(x^n) = n \log_b(x)$
- $\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$
- $\log_b(b) = 1$
- $\log_b(1) = 0$
- $\log_b(xy) = \log_b(x) + \log_b(y)$
- $\log_b(x/y) = \log_b(x) - \log_b(y)$
- $(a^x)^y = a^{xy}$
- $a^x \cdot a^y = a^{x+y}$
- $a^x/a^y = a^{x-y}$
- $b^{\log_b(x)} = x$

Trigonometric Facts.

- $\sin^2(x) + \cos^2(x) = 1$
- $\sin(0) = 0$
- $\sin(\pi/6) = 1/2$
- $\sin(\pi/3) = \sqrt{3}/2$
- $\sin(\pi/2) = 1$
- $\cos(0) = 1$
- $\cos(\pi/6) = \sqrt{3}/2$
- $\cos(\pi/3) = 1/2$
- $\cos(\pi/2) = 0$
- $\sin(\pi/4) = \cos(\pi/4) = \sqrt{2}/2$

Derivatives.

- $\frac{d}{dx} x^n = nx^{n-1}$, for any real number $n$
- $\frac{d}{dx} e^x = e^x$
- $\frac{d}{dx} \ln(x) = \frac{1}{x}$
- $\frac{d}{dx} \sin(x) = \cos(x)$
- $\frac{d}{dx} \cos(x) = - \sin(x)$
• \( \frac{d}{dx} \tan(x) = \sec^2(x) \)

Also keep in mind the following rules for functions \( f \) and \( g \):

- Chain rule: \( \frac{d}{dx}[f(g(x))] = f'(g(x))g'(x) \)
- Sum rule: \( \frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x) \)
- Product rule: \( \frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + f'(x)g(x) \)
- Quotient rule: \( \frac{d}{dx}\left[ \frac{f(x)}{g(x)} \right] = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2} \)

**Integrals.**

- \( \int x^n \, dx = \frac{1}{n+1}x^{n+1} + C \) for \( n \neq -1 \)
- \( \int e^x \, dx = e^x + C \)
- \( \int \frac{1}{x} \, dx = \ln |x| + C \)
- \( \int \sin(x) \, dx = -\cos(x) + C \)
- \( \int \cos(x) \, dx = \sin(x) + C \)

You should be comfortable with performing \( u \)-substitutions and the Fundamental Theorem of Calculus: If \( F \) is an antiderivative for \( f \) then \( \int_a^b f(x) \, dx = F(b) - F(a) \).

**Examples.**

1. Compute the derivative of \( \sin(x)e^{2x} \).

   This involves a combination of the chain rule and the product rule, and the result is
   \[ 2\sin(x)e^{2x} + \cos(x)e^{2x}. \]

2. Compute the derivative of \( \sqrt{\ln(x^3 - 7x + 5)} \).

   This involves a few applications of the chain rule and the power rule. The result is
   \[ \frac{3x^2 - 7}{2(x^3 - 7x + 5)^{\frac{1}{2}}} \]

3. Compute \( \int \sin(3x) \, dx \).

   The result is
   \[ \frac{-1}{3} \cos(3x) + C. \]

4. Compute \( \int_1^3 \frac{4x-6}{x^2-3x+5} \, dx \).

   Recognizing that numerator is simply twice the derivative of the denominator and performing a \( u \)-substitution, we have:
   \[ 2 \ln(x^2 - 3x + 5)|_1^3 = 2 \ln(11) - 2 \ln(3). \]

The front and back cover of your textbook contain many more derivatives and integrals which may be useful at some point during the course.