This exam contains 5 questions. Be sure to write your name, PID, and section on the front of your blue book. This is a closed note, closed book exam. Calculators are not allowed. Show all of your work. No credit will be given for unsupported answers, even if correct. Simplify answers as much as possible.

The second derivative test and general form of the chain rule may be useful at some point on this exam. For your convenience, I have reprinted them here. It is your responsibility to know where and when it makes sense to use this information.

Let $D(x, y) = f_{xx}(x, y)f_{yy}(x, y) - f_{xy}^2(x, y)$.

- If $D(x_0, y_0) > 0$
  - If $f_{xx}(x_0, y_0) > 0$ then $(x_0, y_0)$ is a local minimum.
  - If $f_{xx}(x_0, y_0) < 0$ then $(x_0, y_0)$ is a local maximum.

- If $D(x_0, y_0) < 0$ then $(x_0, y_0)$ is a saddle point.

- If $D(x_0, y_0) = 0$ then the test is inconclusive.

Suppose $f : \mathbb{R}^m \to \mathbb{R}^p$ and $g : \mathbb{R}^n \to \mathbb{R}^m$ and let $Df$ denote the matrix of partial derivatives of $f$. If $g$ is differentiable at $x_0$ and $f$ is differentiable at $g(x_0)$ then $f \circ g$ is differentiable at $x_0$ and we have:

$$D(f \circ g)(x_0) = Df(g(x_0))Dg(x_0).$$