REVIEW PROBLEMS TO PONDER

The following is a list of some questions to get you thinking about important concepts from the course. They are not assigned and you are not required to turn in any solutions. They are purely for you to think about if you wish. Solutions will not be posted since many of these questions have more than one answer; however, you should feel free to come to office hours to discuss them.

1. Suppose you know \( u \cdot v \), but not \( u \) and \( v \) themselves. What information can you recover about \( u \) and \( v \)?

2. Suppose you know \( u \times v \), but not \( u \) and \( v \) themselves. What information can you recover about \( u \) and \( v \)?

3. Given a function \( f : \mathbb{R}^2 \rightarrow \mathbb{R} \), what does \( f_x(a, b) \) tell you?

4. Describe all the techniques you can think of for computing limits.

5. Suppose \( f : \mathbb{R}^2 \rightarrow \mathbb{R} \) and \( f \) has a saddle point at the origin. What might the level curves of \( f \) look like near \((0,0)\)?

6. Parametrize the line segment from the point \((0,1)\) to \((3,5)\) in three different ways.

7. In what direction direction does the gradient vector of a function point, and why? What about the negative gradient?

8. What is the directional derivative of a function in a direction orthogonal to the gradient?

9. How could you explain what a directional derivative is to someone who has taken 20B, but not 20C?

10. Why do we need to normalize the direction vector when computing a directional derivative?

11. When can you apply the second derivative test?

12. What sorts of things should you watch out for when solving a Lagrange Multipliers problem? Try to write your own Lagrange problem which contains some of these tricky features, but can also be solved without a calculator. Can you ensure all critical points have small integer components? These are the sorts of things I think about when writing an exam....

13. Given a path, how do you compute speed, velocity, and acceleration?

14. Given a function for the acceleration vector of a particle, what additional information do you need to compute a function for the position vector?

15. What is the relationship between the gradient of a function and level sets of a function?
(16) How do you set up limits of integration when integrating a function over a 2D y-simple region?

(17) When should you switch the order of integration in a double integral problem?

(18) Is it possible to interchange limits of integration in \( \int \int_D f(x, y) dy \, dx \) if \( D \) is not \( x \)-simple?

(19) Try to think of a way to explain to someone who missed class how to set up limits of integration for a typical triple integral problem to compute the volume of a solid.

(20) What equation describes the tangent plane to a function of two variables at a point? What equation describes the tangent plane to a level set of a function of three variables at a point? How would you write an equation of the tangent line to a level curve of a function of two variables at a point?