Name: ______________________________ **PID:** ________________

TA: _______________________ **Sec. No:** _____ **Sec. Time:** _____

Math 20A.
Final Examination
December 10, 2009

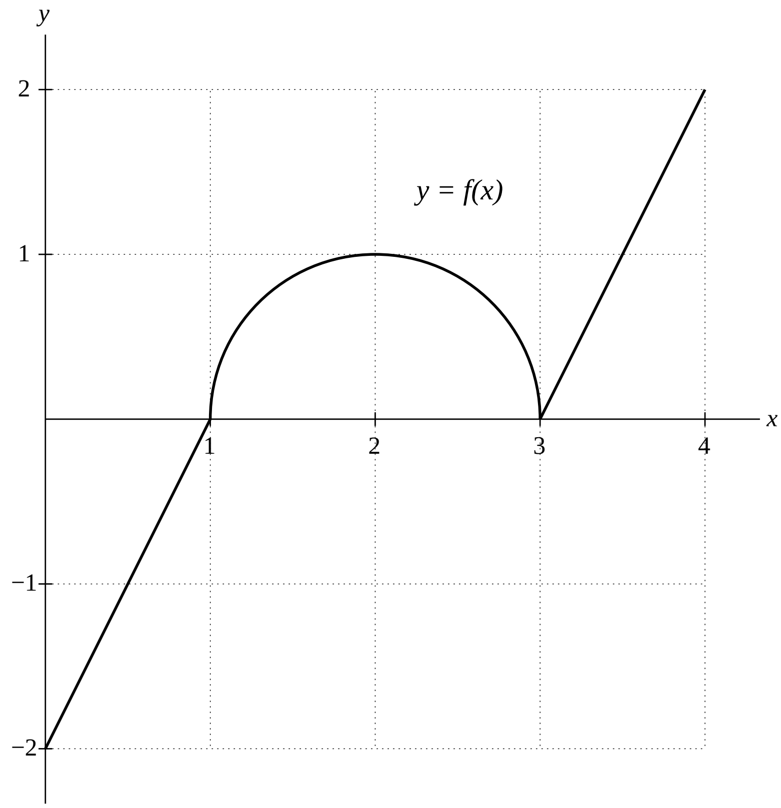
Turn off and put away your cell phone.
No calculators or any other electronic devices are allowed during this exam.
You may use one page of notes, but no books or other assistance during this exam.
Read each question carefully, and answer each question completely.
Show all of your work; no credit will be given for unsupported answers.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
If any question is not clear, ask for clarification.

#	Points	Score
1	6	
2	6	
3	6	
4	6	
5	8	
6	6	
7	6	
8	8	
9	8	
Σ	60	

1. (6 points) Use the intermediate value theorem to show that the equation $\cos(2x) = x$ has a solution in the interval $\left(0, \frac{\pi}{4}\right)$.

2. (6 points) Find the slope of the line tangent to the curve $x^y = y^x$ at the point $(2, 4)$.

3. (6 points) Let $f(x)$ be a function defined over $[0, 4]$ whose graph is shown below. The graph of $f(x)$ over $[1, 3]$ is a semicircle centered at $(2, 0)$ with radius 1, and the other parts of the graph are straight lines.



(a) (6 points) Determine $\displaystyle\int_0^4 f(x)\,dx$ geometrically.

(b) Let $F(x) = \int_0^x f(t)\,dt$.

i. Find $F'(2)$.

ii. Find $F'(3)$.

4. (6 points) A spherical weather balloon is being inflated at the rate of 12 cubic inches per second. What is the radius of the balloon when its surface area is increasing at a rate of 8 square inches per second?

   Note: The formulas for the volume V and surface area A of a sphere of radius r are $V = \frac{4}{3}\pi r^3$ and $A = 4\pi r^2$.

5. (6 points) Find the values of a and b for which the function

$$f(x) = \begin{cases} ax^2 + bx + 2 & \text{if } x \leq 1, \\ -ax^4 - bx^2 & \text{if } x > 1. \end{cases}$$

is differentiable for all real numbers x.

6. (8 points) Let $f(x) = x^3 - 27x + 5$

(a) Find the interval(s) where f is increasing and the intervals where f is decreasing.

(b) Find the local maximum and local minimum value(s) of f.

(c) Find the intervals where the graph of f is concave up and the intervals where the graph of f is concave down.

(d) Determine the inflection points of the graph of f.

7. (6 points) Evaluate the following limits.

(a) $\displaystyle\lim_{x\to 0} \frac{\tan(\pi x)}{\ln(1+x)}$

(b) $\displaystyle\lim_{x\to 0} x^2 \ln|x|$

8. (8 points) Compute the derivatives of the following functions.

(a) $f(x) = (x^4 - 3x^2 + 6)^3$

(b) $f(x) = \dfrac{x}{3 - x^2}$

(c) $f(x) = 6x(x^2 - 6)^{\frac{1}{3}} + \pi^2$

(d) $f(x) = \displaystyle\int_0^x \sqrt{3 + t^3}\, dt$

9. (8 points) A triangle in the first quadrant is formed by the x and y axes and a line passing through the point $(3, 8)$.

   (a) What is the minimum possible area for such a triangle?

   (b) What line gives this minimum area?