Quiz 1 Solution for MATH 20C, 2009

1. *For* $R = (1, 4, 3)$, *find the point* P *such that* $\mathbf{v} = \overrightarrow{PR}$ *has components* $< 3, -2, 3 >$ *and sketch* $\mathbf{v}$.

Ans. Let $P = (a, b, c)$. Then

$$\mathbf{v} = \overrightarrow{PR} =< 1 - a, 4 - b, 3 - c >=< 3, -2, 3 >$$

$$\Rightarrow 1 - a = 3, \quad 4 - b = -2, \quad 3 - c = 3$$

So,

$$a = -2, \quad b = 6, \quad c = 0$$

$$P = (-2, 6, 0).$$

2. Find a vector parametrization for the line which passes through $(-2, 0, -2)$ and $(4, 3, 7)$.

Ans. The direction vector of the line is (or is parallel to)

$$\mathbf{v} =< 4 - (-2), 3 - 0, 7 - (-2) >=< 6, 3, 9 > .$$

(You could take the direction vector $< 2, 1, 3 >$). Then the line equation is

$$r(t) =< -2, 0, -2 > +t < 6, 3, 9 >=< -2 + 6t, 3t, -2 + 9t >$$

So, the vector parametrization for the given line is

$$\begin{cases} x = -2 + 6t \\ y = 3t \\ z = -2 + 9t \end{cases} \quad \text{or} \quad \begin{cases} x = -2 + 2t \\ y = t \\ z = -2 + 3t \end{cases}$$

3. Determine whether the two vectors are orthogonal and if not, find the cosine of the angle between the vectors.

$$< 1, 1, 5 >, \quad < 1, -1, 5 >$$

Ans. To check the orthogonality of the two vectors, we check if the dot product of the two vectors is zero or not ;

$$< 1, 1, 5 > \bullet < 1, -1, 5 >= 1 - 1 + 25 = 25 \neq 0.$$

Since the dot product is nonzero, the given two vectors are **not** orthogonal.

To calculate the cosine of the angle between the vectors, we note that

$$v \bullet w = ||v|| \cdot ||w|| \cos\theta.$$

$$\Rightarrow 25 = \sqrt{1+1+25} \cdot \sqrt{1+1+25} \cos\theta,$$

where θ is the angle between the two given vectors. Hence,

$$\cos\theta = \frac{25}{27}.$$