

Quiz 2 Solution for MATH 20C, 2009

1. Consider three points $P = (1, 1, 5)$, $Q = (3, 4, 3)$, $R = (1, 5, 7)$ in three space.

(a) Use the cross product to find the area of the triangle with vertices P , Q and R .

Ans. $\overrightarrow{PQ} = \langle 2, 3, -2 \rangle$, $\overrightarrow{PR} = \langle 0, 4, 2 \rangle$.

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} i & j & k \\ 2 & 3 & -2 \\ 0 & 4 & 2 \end{vmatrix} = 14i - 4j + 8k.$$

$$\text{Area of } \triangle ABC = \frac{\|\overrightarrow{PQ} \times \overrightarrow{PR}\|}{2} = \frac{\sqrt{196 + 16 + 64}}{2} = \frac{\sqrt{276}}{2} = \sqrt{69}.$$

(b) Find the equation of the plane passing through the three points P , Q and R .

Ans. We can take the normal vector of the plane as $\vec{n} = \overrightarrow{PQ} \times \overrightarrow{PR}$, so the plane equation is

$$14(x - 1) - 4(y - 1) + 8(z - 5) = 0$$

$$\text{or, } 7x - 2y + 4z = 25.$$

2. We define the angle between two planes by the angle between their normal vectors. Compute the angle between two planes $x + z = 3$ and $x - y - z = 5$.

Ans. The normal vectors of the given planes are $\vec{n}_1 = \langle 1, 0, 1 \rangle$ and $\vec{n}_2 = \langle 1, -1, -1 \rangle$. We take the dot product of the normal vectors to get the cosine of the angle between two planes ; $\vec{n}_1 \bullet \vec{n}_2 = 1 - 1 = 0$. Since the dot product is zero, they are orthogonal, i.e., the angle between two planes is $\pi/2$.

3. Find the parametrization of the intersection curve of the plane $y = \frac{1}{2}$ and the sphere $x^2 + y^2 + z^2 = 1$.

Ans. Since y is given by $1/2$, the second surface becomes $x^2 + z^2 = 3/4$ which is an equation of a circle centered at the origin with radius $\sqrt{3}/2$. So we parametrize the circle by $x = \frac{\sqrt{3}}{2} \cos t$, $z = \frac{\sqrt{3}}{2} \sin t$ and so, the parametrization of the intersection curve is

$$r(t) = \left\langle \frac{\sqrt{3}}{2} \cos t, \frac{1}{2}, \frac{\sqrt{3}}{2} \sin t \right\rangle.$$