Quiz 4 Solution for MATH 20C, 2009

1. Evaluate the limit or determine that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{y^2}{x^2+y^2}$$

Ans. We choose two different path approaching $(0,0)$ and find that there exist two limit values along different paths.
First, along the y-axis, i.e., when $x = 0$,

$$\lim_{(0,y)\to(0,0)} \frac{y^2}{y^2} = \lim_{(0,y)\to(0,0)} 1 = 1.$$

Secondly, if we choose the x-axis, i.e., when $y = 0$,

$$\lim_{(x,0)\to(0,0)} \frac{0}{x^2+0} = \lim_{(x,0)\to(0,0)} 0 = 0.$$

We checked that along two different coordinate axes, the limit values are different. Hence, the limit **does not exist**.

2. Compute the derivative indicated.

$$f(x,y) = x\ln(y^2), \quad f_{yy}(2,3)$$

Ans. Note that $f(x,y) = 2x\ln y$. Then the first and second partial derivatives of f with respect to y are

$$f_y(x,y) = \frac{2x}{y}, \quad f_{yy}(x,y) = -\frac{2x}{y^2} \quad \Rightarrow \quad f_{yy}(2,3) = -\frac{4}{9}$$

3. Use linear approximation to $f(x,y) = \sqrt{x/y}$ at $(9,4)$ to estimate $\sqrt{9.1/3.9}$.

Ans. Note that the linear approximation is defined by the value on the tangent plane : at $(9,4)$,

$$L(x,y) = f(9,4) + f_x(9,4)(x-9) + f_y(9,4)(y-4).$$

$$f_x(x,y) = \frac{1}{2\sqrt{xy}}, \quad f_y(x,y) = -\frac{1}{2y}\sqrt{\frac{x}{y}}$$

$$\Rightarrow \quad f_x(9,4) = \frac{1}{2\sqrt{36}} = \frac{1}{12}, \quad f_y(9,4) = -\frac{1}{8}\sqrt{\frac{9}{4}} = -\frac{3}{16}.$$

Using the value $f(9,4) = \sqrt{9/4} = 3/2$, we approximate $\sqrt{9.1/3.9}$ by

$$\sqrt{\frac{9.1}{3.9}} = f(9.1,3.9) \approx L(9.1,3.9) = \frac{3}{2} + \frac{1}{12}(9.1-9) - \frac{3}{16}(3.9-4) = \frac{3}{2} + \frac{1}{12}(0.1) - \frac{3}{16}(-0.1) = \frac{733}{480}$$

Note that $733/480 = 1.52705$.