Quiz 5 Solution for MATH 20C, 2009

1. Let $f(x,y) = xe^{x^2-y}$ and $P = (1,1)$.

(a) Find the rate of change of f in the direction of ∇f_P.

Ans.

$$\nabla f =< e^{x^2-y} + 2x^2e^{x^2-y}, -xe^{x^2-y} >$$

and at $P = (1,1)$, $\nabla f(P) =< 3, -1 >$.

$$D_{\nabla f(P)}f(P) = ||\nabla f(P)|| = \sqrt{3^2+1} = \sqrt{10}.$$

(b) Find the rate of change of f in the direction of a vector making an angle of 45° with ∇f_P.

Ans. Let u be the unit vector making an angle of 45° with ∇f_P, then

$$\begin{aligned} D_u f(P) &= ||\nabla f(P)|| \cos\theta \quad \text{where} \quad \theta = \pi/4 \\ &= \sqrt{10} \cdot \frac{\sqrt{2}}{2} \\ &= \sqrt{5}. \end{aligned}$$

2. Find a vector normal to the surface $3z^3 + x^2y - y^2x = 1$ at $P = (1,-1,1)$.

Ans. Let $F(x,y,z) = 3z^3 + x^2y - y^2x$. Then $\nabla F(P)$ is normal to the level surface $F(x,y,z) = 1$ at P.

$$\nabla F =< 2xy - y^2, x^2 - 2xy, 9z^2 >$$

So, at $P = (1,-1,1)$, the normal vector to the given level surface is

$$\nabla F(P) =< -3, 3, 9 > .$$

3. Calculate $\frac{\partial w}{\partial z}$ where w is implicitly defined by $x^2w + w^3 + wz^2 + 3yz = 0$.

Ans. Let $F(x,y,z,w) = x^2w + w^3 + wz^2 + 3yz$. Then for the given $F(x,y,z,w) = 0$,

$$\frac{\partial w}{\partial z} = -\frac{F_z}{F_w}.$$

$$F_z = 2wz + 3y, \quad F_w = x^2 + 3w^2 + z^2.$$

Hence,

$$\frac{\partial w}{\partial z} = -\frac{2wz + 3y}{x^2 + 3w^2 + z^2}.$$