**Review for Midterm I, MATH 20C, 2009**

# 1 Chapter 12.

**1.** [12.2] Find an equation of a sphere if one of its diameters has end-points $(2, 1, 4)$ and $(4, 3, 10)$.

**2.** [12.2, 12.3, 12.4] Given $P = (2, 1, 3)$, $Q = (1, -1, 0)$, $R = (1, 1, -1)$, $S = (0, 1, -2)$ determine whether the vectors $\overrightarrow{PQ}$ and $\overrightarrow{RS}$
(a) are parallel
(b) are perpendicular
by using the cross and dot products.

**3.** [12.3, 12.4] Let $\mathbf{a} =< 2, 2, 2 >$ and $\mathbf{b} =< 5, c, 5 >$ be vectors, where $c$ is to be determined.
(a) For what value of $c$ is $\mathbf{b} =< 5, c, 5 >$ orthogonal to $\mathbf{a} =< 2, 2, 2 >$?
(b) For what value of $c$ is $\mathbf{b} =< 5, c, 5 >$ parallel to $\mathbf{a} =< 2, 2, 2 >$?

**4.** [12.4] Let $A = (2, 1, -1)$, $B = (3, 0, -2)$, $C = (3, 2, 1)$ and $D = (-2, 0, 1)$.
(a) Find the area of the parallelogram that has $AB$ and $AC$ as adjacent sides.
(b) Find the volume of the parallelepiped that has edges $AB, AC$, and $AD$.

**5.** Let $\mathbf{a} =< 1, 1, 1 >$ and $\mathbf{b} =< -2, -2, 1 >$ .
(a) [12.4] Find two vectors that are orthogonal to both $\mathbf{a}$ and $\mathbf{b}$.
(b) [12.4] Find the sine of the angle between $\mathbf{a}$ and $\mathbf{b}$.

**6.** [12.4, 12.5] Given the points $P = (2, 0, 0)$, $Q = (0, 1, 0)$ and $R = (0, 0, -5)$.
(a) Find a vector that is normal to the plane through $P$, $Q$ and $R$.
(b) At which point does the line $x = 1 + t$, $y = 1 - t$, $z = -3 - 3t$ intersect the plane?

**7.** [12.4, 12.5] Let $P(1, 0, -3)$, $Q(0, -2, -4)$, and $R(4, 1, 6)$ be points.
(a) Find the equation of the plane through the points $P$, $Q$ and $R$.
(b) Find the area of the triangle with vertices $P$, $Q$ and $R$.

**8.** [12.4, 12.5] (a) Find a unit vector which is orthogonal to $< 1, -1, -1 >$ and $< 1, 1, 2 >$.
(b) Find the line of intersection of the two planes

$$x - y - z = 2, \quad x + y + 2z = -2.$$

**9.** [12.5] Find an equation for the plane containing the lines

$$\lambda(t) =< 1, 3, 5 > +t < 1, 4, 7 > \quad \text{and} \quad \mu(t) =< 1, 3, 5 > +t < 2, -1, 4 > .$$

**10.** [12.5] (a) Find the equation of the plane which goes through the points $(1, 1, -1)$, $(0, 1, 0)$ and $(1, -1, 0)$.
(b) Compute the perpendicular distance from the point $(1, 1, 1)$ to the plane described in part (a).

**11.** [12.5] (a) Show that the four points $(2, 0, -3)$, $(0, 5, 4)$, $(1, 1, -1)$ and $(5, -12, -18)$ lie in a plane.
(b) Find the distance between this plane and the parallel plane $x - y + z = 2$.
(*Hint: Find the distance between a point on one plane and the other plane.*)

## 2 Chapter 13.

**1.** A particle's position functions is $\mathbf{r}(t) =< 2\cos t, 2\sin t, 2t >$ for $0 \le t \le 2\pi$.
(a) [13.1, 13.2] Find the particle's velocity $\mathbf{v}(t)$ and speed $|\mathbf{v}(t)|$ as a function of time $t$.
(b) [13.2] Find the particle's acceleration $\mathbf{a}(t)$ as a functions of time $t$.
(c) [12.3, 13.2] Find the angle between the particle's position $\mathbf{r}(t)$ and acceleration $\mathbf{a}(t)$ as a function of time $t$.
(d) [13.3] Determine how far the particle traveled during the time interval $0 \le t \le 2\pi$.

**2.** [13.1, 13.2] Consider the curve defined parametrically by the equations

$$x(\theta) = 2\theta - \pi\sin\theta, \quad y(\theta) = 2 - \pi\cos\theta$$

for $-\pi \le \theta\pi$.
(a) Find the point where the curve intersects itself.
(b) Write down the equation for each line tangent to the curve at the point you found in part (a).

**3.** [13.1, 13.2] Consider the curve defined by the parametric equations

$$x = t, \quad y = 2t^{3/2}, \quad \text{where} \quad 0 \le t \le 2.$$

(a) Write down the equation for the line that is tangent to the curve when $t = 1$.
(b) Find the arc length of the curve.

**4.** [13.2] Consider the curve $C$ given by the parametric equations

$$x = \cos t, \quad t = 2 + \sin t, \quad \text{where} \quad 0 \le t \le 2\pi.$$

Find the points where the tangent to the curve is vertical.

**5.** [13.2] The curve is given by $x = \sin t$, $y = 1 + \sin t\cos t$ has two tangents at the point $(0, 1)$.
(a) Find the tangents.
(b) Find the angle between them.

**6.** [13.2] Consider the curve $C$ given by $x = \frac{1}{2}\cos 2t$, $y = 1 + \sin t\cos t$, $t \in \left[0, \frac{\pi}{2}\right]$.
Determine if the tangent of $C$ is horizontal at any point (and find the point).

**7.** Let $r(t) =< 3t + 1, 4t - 5, 2t >$.
(a) Calculate $s(t) = \int_0^t ||r'(u)||du$ as a function of $t$.
(b) Find the inverse $\varphi(s) = t(s)$ and show that $r_1(s) = r(\varphi(s))$ is an arc length parametrization.