Review for Midterm II, MATH 20C, 2009

1 Chapter 13

1. A parking ramp follows the curve in space traced by the function

$$r(t) =< \cos t, \sin t, t > \quad \text{for} \quad 0 < t < 4\pi.$$

What is the curvature function $\kappa(t)$ of the ramp?

2. (13.5) Santa Claus' sleigh loses power after taking off at 64 ft/s from a 32 foot high roof at an angle 45°. How many seconds elapse before he hits the ground?
(For this problem, assume that Santa exists and that his magic is not powerful enough to alter the earth's gravitational acceleration of 32 ft/s^2).

3. (13.5) At $t = 0$, an airplane takes off. At that moment, its position vector is $< 0, 0, 0 >$ and its velocity vector is $< 1, 2, 0 >$. Find its position vector at time $t = 6$, if the acceleration of the airplane is $a(t) =< 1, 0, t >$.

2 Chapter 14

1. (14.2) Compute the following limits. If the limit does not exist, explain why.

$$\text{(a)} \quad \lim_{(x,y)\to(2,3)} \frac{x^2 + xy + 2y^2 - 1}{x^2 - y^2 + 4}$$

$$\text{(b)} \quad \lim_{(x,y)\to(0,0)} \frac{x^2 + y^2}{x^2 + xy + y^2}$$

2. (14.3) Let $f(x, y, z) = e^{xy^2} + \ln(y + z^3)$. Compute the following partial derivatives of f.

(a) f_x
(b) f_y
(c) f_{zx}
(d) f_{zy}

3. (14.4) Let $F(x, y, z) = z - y^2 + x^2 + 3$. Find the tangent plane to the level surface $F(x, y, z) = 0$ at the point $(1, 2, 0)$.

4. (14.4) Consider the surface given by $z = f(x, y) = \sqrt{x^2 + 3y^2}$.

(a) Find the tangent plane to the surface at the point $(1, 1, 2)$.
(b) A student was asked to find an approximation for $f(1.1, 1.2)$ but the professor did not allow calculators. The student noticed that $f(1.1, 1.2)$ is approximately $f(1, 1) = \sqrt{1 + 3} = 2$. Use the linear approximation to get a better approximation.

5. (14.4) The tangent plane to a surface $z = f(x, y)$ at the point $(-2, 3, 4)$ has equation $4x+2y+z = 2$. Estimate $f(-2.1, 3.1)$.

6. (14.4) Approximate $(\sqrt[3]{28})^2 + (\sqrt{24})^3$.

7. (14.4) The dimensions of a closed rectangular box are measured as 20 cm, 10 cm, and 5 cm respectively, with a possible error of 0.2 cm in each dimension. Use differentials to estimate the maximum error in calculating the volume of the box.

8. (14.5) Let $f(x, y) = x^2 + 6y^2$.

   (a) Find the unit vector in the direction for which the directional derivative of f at the point $(-3, 4)$ is maximum.
   (b) Find the unit vectors in the directions for which the directional derivative of f at the point $(-3, 4)$ is zero.
   (c) Compute the directional derivative of f at the point $(-3, 4)$ in the direction toward the origin.

9. (14.5) Suppose that $f(x, y)$ is a function of two variables and that the directional derivative of f in the direction $\mathbf{u} =< 1, 0 >$ at the point $(2, 3)$ is 5, i.e., $D_{\mathbf{u}}f(2, 3) = 5$, and that $D_{\mathbf{v}}f(2, 3) = -2$, if $\mathbf{v} =< 0, 1 >$.

   (a) What is the gradient of f at $< 2, 3 >$, i.e., $\nabla f(2, 3)$?
   (b) In which direction $\mathbf{w}$ is the directional derivative $D_{\mathbf{w}}f(2, 3) = 0$?

10. (14.5) Let $F(x, y) = x^3 - x^2 + y^2 - y + 1$.

    (a) Find the gradient $\nabla F(x, y)$.
    (b) Find the directional derivative in the direction of $< 1, 2 >$ at the point $(2, 3)$.
    (c) Let $\mathbf{r}(t) =< f(t), g(t) >$ be a curve such that $\mathbf{r}(0) =< 1, 0 >$ and $\mathbf{r}'(0) =< 1, 1 >$. Let $h(t) = F(\mathbf{r}(t))$. Find $h'(0)$.

11. (14.6) Evaluate $\frac{\partial h}{\partial q}$ at $(q, r) = (3, 2)$, where $h(u, v) = ue^v$, $u = q^3$, $v = qr^2$.

12. (14.6) Evaluate $\frac{\partial z}{\partial y}$ when $e^{xy} + \sin(xz) + y = 0$.