Relevant readings: Cutland Ch 9

1. (Cutland 9.1.7.3-4)
   (a) Show that \( \{ x : \phi_x \text{ is total} \} \leq_m \{ x : W_x \text{ is infinite} \} \).
   (c) Show that neither of the above sets is \( m \)-reducible to an r.e. set.

2. (Cutland 9.2.9.5) Let \( a, b \) be \( m \)-degrees.
   (a) Show that the least upper bound of \( a, b \) is uniquely determined; denote it by \( a \cup b \).
   (b) Show that if \( a \leq_m b \) then \( a \cup b = b \).
   (c) Show that if \( a, b \) are both r.e. then so is \( a \cup b \).
   (d) Let \( A \in a \) and let \( a^* \) denote \( d_m(\bar{A}) \). Show that \( (a \cup a^*)^* = a \cup a^* \).

3. (Cutland 9.4.10.5a) Prove that for any set \( A \) and (oracle) set \( B \),
   \[ A \text{ is } B\text{-recursive} \iff A, \bar{A} \text{ are both } B\text{-r.e.} \]

4. (Cutland 9.4.10.8) Let \( A \) be any set. Show that for any set \( B \),
   \[ B \text{ is } A \text{-r.e.} \iff B \leq_m K^A. \]

5. (Cutland 9.5.21.2) Prove that for any sets \( A, B \),
   \[ A \leq_T B \iff K^A \leq_m K^B, \]
   and
   \[ A =_T B \iff K^A =_m K^B. \]