1. In example 3.5.13, Cutland gives a Post system computing the function \( x \mapsto x^2 \) using the alphabet \( \{1, \cdot\} \). Find a Post system to compute the function \( x \mapsto x^3 \) with the same alphabet. Hint: you may want to use auxiliary symbols in the generating system (see Definitions 5.7 and 5.12).

2. (Cutland 2.4.16) Let \( \pi(x, y) = 2^x(2y + 1) - 1 \). Show that \( \pi \) is a primitive recursive bijection \( \mathbb{N}^2 \to \mathbb{N} \), and that the functions \( \pi_1, \pi_2 \) such that \( \pi(\pi_1(z), \pi_2(z)) = z \) for all \( z \) are also primitive recursive.

3. Let \( A \) be any finite subset of \( \mathbb{N} \). Prove that the characteristic function of \( A \) is primitive recursive.

4. (Cutland 2.5.4 (1)) Suppose that \( f(x) \) is a total injective URM computable function. Prove that \( f^{-1} \) is URM computable.

5. Prove directly that \( \mathcal{T} \subseteq \mathcal{C} \). That is, given a TM \( M \), define a URM program \( P \) which computes the same functions as \( M \). Some hints:

   - Use registers to store a code for the symbol under the reading head, a number coding the symbols on the tape to the left of the reading head, and a number coding the symbols on the tape to the right of the reading head.
   - Design a block of instructions for each internal state \( q \) of \( M \).
   - For each such \( q \), branch based on symbol being read while in that state. There will be several cases to consider.