MATH 154 Homework 4
Due November 21, 2012
Version November 12, 2012

Hand in: AP&M 6436 (not in drop-off box)

Recommended practice:

HHM 2.4 pp. 154-155 # 2,3,13
HHM 2.5 pp. 161-162 # 2,3,4,5
Ver Part 1 pp. 16 #1, 3, 8
HHM 1.6.1 pp. 87-88 #1,2,3

Assigned questions to hand in:

(1) Determine the number of sequences \( \langle x_1, \ldots, x_k \rangle \) with each \( x_i \in \{1, \ldots, n\} \) with the given restriction: For \( i \leq n \), \( x_i \neq x_{i+1} \).

Ver Part 1 pp. 16 #1 b

(2) Let \( A \) be the set of all sequences of positive integers (of any length) which add up to \( n \), and let \( B \) be the set of all subsets of \( \{1, \ldots, n-1\} \). Find a bijection \( f : A \rightarrow B \). Deduce that \( |A| = 2^{n-1} \) for \( n \geq 1 \).

Ver Part 1 pp. 16 # 7

(3) Find the smallest value of \( m \) so that the following statement is valid: Any collection of \( m \) distinct positive integers must contain at least two numbers whose sum or difference is a multiple of 10. Prove that your value is best possible.

HHM 2.4.5 p. 156

(4) A noted vexillologist tells you that 30 of the 50 U.S. state flags have blue as a background color, twelve have stripes, 26 exhibit a plant or animal, nine have both blue in the background and stripes, 23 have both blue in the background and feature a plant or animal, and three have both stripes and a plant or animal. One of the flags in this last category (California) does not have any blue in the background. How many state flags have no blue in the background, no stripes, and no plant or animal featured?

HHM 2.5.1 p. 161

(5) Use Theorem 2.6 (Inclusion-Exclusion Principle) to determine the chromatic polynomial for the yield sign (add a single edge to the bipartite graph \( K_{1,3} \)).

HHM 2.5.10a p. 163