109 Spring 2011 - Practice Problems for Midterm

Some of the problems on the test may be very similar to these. On the other hand, some of these problems are harder than what will appear on the test. Solutions will **not** be distributed for these practice problems (other than the ones already available for homework). It is often much more productive to figure out for yourself how to **start** a problem and then ask for help if and when you get stuck.

Exercise. All the homework exercises: both those that you handed in and those that were recommended but not collected. Also, some examples from lectures or section were taken from the textbook. You should make sure to review them as well. That is:

- Chapter 1: 1, 2
- Chapter 2: 1, 3, 5
- Chapter 3: 1, 2, 3, 6, 8i
- Chapter 4: 2, 3, 5, 7
- Chapter 5: 1, 2, 4
- Part I Problems: 1, 2, 3, 4, 5, 7, 9, 10, 11, 13.
- Chapter 6: 1, 2, 5, 6
- Chapter 7: 1, 2(i, iii, v), 4 (iii, iv), 7
- Chapter 8: 1, 3, 4
- Chapter 9: 1 (i, ii), 4, 5,
- Chapter 10: 1, 3
- Chapter 11: 1, 2, 5 (i,ii)
- Part II Problems: 2, 4, 10, 11, 12, 14, 16, 17, 18
- Limits and continuity homework and section work.

Exercise. Prove that for any statements P and Q, if $(P \implies Q) \implies P$ is true then P is true.

Exercise. Which of the following statements are true? Prove those that are true and disprove those that are false.

(a) For all integers n, there exists an integer q such that $n = 2q$.
(b) For all integers q, there exists an integer n such that $n = 2q$.
(c) There exists a natural number n such that for all natural numbers p, we have $n \leq p$.
(d) There exists a natural number p such that for all natural numbers n, we have $n \leq p$.
(e) For all natural numbers n, there exists a natural number p, such that $n < p$.
(f) For all natural numbers p, there exists a natural number n, such that $n < p$.

Exercise. What is the negation of each of the following propositions?

(a) n is not a perfect square and n is divisible by 3.
(b) For every n in $\mathbb{Z}^{\geq}$ there exist p and q (each in $\mathbb{Z}^{\geq}$) such that $p^2 = nq^2$.

Exercise. What is the contrapositive of each of the following propositions?

(a) If n is divisible by 2 then n is not divisible by 6.
(b) If n is divisible by 6 then n is divisible by 2.

Is either of the above propositions true? Is either one false? Justify your answer.

Exercise. Prove that $\lim_{x \to 3}(5x - 1) = 14$.

Exercise. Prove that the sequence given by $f(n) = \frac{1}{n!}$ is a null sequence.

Exercise. Prove that if $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} f(x) = M$ then $L = M$.

Hint: Suppose for a contradiction that $L \neq M$. Then take $\epsilon = \frac{1}{2}|L - M|$ in the definitions of the two limits.

Exercise. Prove by induction that for any natural number n and any real numbers x and y, we have $x^n y^n = (xy)^n$.

Exercise. Show that postage of 24 cents or more can be achieved by using only 5-cent and 7-cent stamps.

Hint: induction might be helpful.

Exercise. Let A, B, and C be sets. Which of the following is **false** in general?

(a) $A \subseteq A \cup B \cup C$
(b) $A \cap B \cap C \subseteq A$
(c) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
(d) $A \cap (B \cup C) = (A \cup B) \cap (A \cup C)$

Exercise. Prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

Exercise. If A and B are nonempty sets, show that $A \times B = B \times A$ implies $A = B$.

Exercise. Fill in the blank and then prove: For two finite sets A, B there is a ________________ between A and B if and only if $|A| = |B|$.

Exercise. Prove that if A and B are finite sets with $B \subseteq A$ then $|A - B| = |A| - |B|$.

Exercise. For each of the following functions, indicate if it is injective or surjective (or both).

(a) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^4$.
(b) $f : \mathbb{R}^{\geq}\mathbb{R}$ defined by $f(x) = x^4$.
(c) $f : \mathbb{R}^+ \to \{x \in \mathbb{R} : 0 < x < 1\}$ defined by $f(x) = \frac{1}{1+x}$.