109 Spring 2011 - Quantifiers and Functions

Exercise (II.11)**.** Give a proof or a counterexample for each of the following statements.

(i) $\forall x \in \mathbb{R} \exists y \in \mathbb{R} (x + y > 0)$
(ii) $\forall x \in \mathbb{R} \exists y \in \mathbb{R} (x - y > 0)$
(iii) $\exists x \in \mathbb{R} \forall y \in \mathbb{R} (x + y > 0)$
(iv) $\forall x \in \mathbb{R} \exists y \in \mathbb{R} (xy > 0)$
(v) $\exists x \in \mathbb{R} \forall y \in \mathbb{R} (xy > 0)$
(vi) $\forall x \in \mathbb{R} \exists y \in \mathbb{R} (xy \geq 0)$
(vii) $\exists x \in \mathbb{R} \forall y \in \mathbb{R} (xy \geq 0)$
(viii) $\forall x \in \mathbb{R} \exists y \in \mathbb{R} (x + y > 0 \text{ or } x + y = 0)$
(ix) $\forall x \in \mathbb{R} \exists y \in \mathbb{R} (x + y > 0 \text{ and } x + y = 0)$
(x) $\big(\forall x \in \mathbb{R} \exists y \in \mathbb{R} (x + y > 0)\big)$ and $\big(\forall x \in \mathbb{R} \exists y \in \mathbb{R} (x + y = 0)\big)$

Exercise (II.14)**.** Define functions f and $g : \mathbb{R} \to \mathbb{R}$ by $f(x) = x^2$ and $g(x) = x^2 - 1$. Find the functions $f \circ f$, $f \circ g$, $g \circ f$, and $g \circ g$. List the elements of the set $\{x \in \mathbb{R} : fg(x) = gf(x)\}$.