

109 Spring 2011 - Implication and Subsets

Exercise (I.4). Prove the following statements concerning positive integers a, b , and c .

(i) $(a \text{ divides } b) \text{ and } (a \text{ divides } c) \implies a \text{ divides } (b + c)$.

(ii) $(a \text{ divides } b) \text{ or } (a \text{ divides } c) \implies a \text{ divides } bc$.

Exercise (I.5). Which of the following conditions are *necessary* for the positive integer n to be divisible by 6? Which are sufficient?

- (i) 3 divides n
- (ii) 9 divides n
- (iii) 12 divides n
- (iv) $n = 12$

Exercise (II.10). We define half-infinite intervals as follows

$$(a, \infty) = \{x \in \mathbb{R} \mid x > a\};$$

$$[a, \infty) = \{x \in \mathbb{R} \mid x \geq a\}.$$

Prove that

$$(i) \quad (a, \infty) \subseteq [b, \infty) \iff a \geq b,$$

$$(ii) \quad [a, \infty) \subseteq (b, \infty) \iff a > b,$$