

109 Spring 2011 - Injections and Surjections

Exercise (II.12). Suppose that $A \subseteq \mathbb{Z}$. Write the following statement entirely in symbols using the quantifiers \forall and \exists . Write out the negative of this statement in symbols.

There is a greatest number in the set A .

Give an example of a set A for which this statement is true. Give an example of a set A for which it is false.

Exercise (II.17). Functions $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ are defined as follows.

$$f(x) = \begin{cases} x + 2 & \text{if } x < -1, \\ -x & \text{if } -1 \leq x \leq 1 \\ x - 2 & \text{if } x > 1; \end{cases} \quad g(x) = \begin{cases} x - 2 & \text{if } x < -1, \\ -x & \text{if } -1 \leq x \leq 1 \\ x + 2 & \text{if } x > 1 \end{cases}$$

Find the functions $f \circ g$, $g \circ f$. Is g the inverse of f ? Is f injective or surjective? Is g ?

Exercise (II.18).

(a) Suppose that $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ are surjections. Prove that the composite $g \circ f : X \rightarrow Z$ is a surjection.

(b) Suppose that $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ are injections. Prove that the composite $g \circ f : X \rightarrow Z$ is an injection.