109 Spring 2011 - Limits and Functions

Exercise. We say that a sequence $f : \mathbb{Z}^+ \to \mathbb{R}$ is **bounded** if there is some number $b \in \mathbb{R}$ such that for each $n \in \mathbb{Z}^+$,

$$|f(n)| \leq b.$$

(a) Prove that the harmonic sequence is bounded.

(b) Prove that the sequence of (positive) powers of 2 is not bounded.

Exercise (Hard!)**.** Prove that if a sequence converges to a finite limit then it is bounded. Is the converse true?

Exercise (II.16)**.** Determine which of the following functions $f_i : \mathbb{R} \to \mathbb{R}$ are injective, which are surjective, and which are bijective. Write down an inverse function of each of the bijections.

(i) $f_1(x) = x - 1$;
(ii) $f_2(x) = x^3$;
(iii) $f_3(x) = x^3 - x$;
(iv) $f_4(x) = x^3 - 3x^2 + 3x - 1$;
(v) $f_5(x) = e^x$;
(vi) $f_6(x) = \begin{cases} x^2 & \text{if } x \geq 0 \\ -x^2 & \text{if } x \leq -0 \end{cases}$