

109 Spring 2011 - Truth Tables - Solutions

Exercise (I.2). By using truth tables prove that, for all statements P and Q , the three statements

- (1) ' $P \implies Q$ '
- (2) ' $(P \text{ or } Q) \iff Q$ '
- (3) ' $(P \text{ and } Q) \iff P$ '

are equivalent.

Recall that \iff is defined by the truth table

P	Q	$P \iff Q$
T	T	T
T	F	F
F	T	F
F	F	T

Therefore, we have:

P	Q	$P \implies Q$	$P \text{ or } Q$	$P \text{ and } Q$	$(P \text{ or } Q) \iff Q$	$(P \text{ and } Q) \iff P$
T	T	T	T	T	T	T
T	F	F	T	F	F	F
F	T	T	T	F	T	T
F	F	T	F	F	T	T

Since the third, sixth, and seventh columns of the truth-table are identical the three statements are equivalent.

Exercise (I.3). Prove that the three basic connectives ‘or’, ‘and’, and ‘not’ can all be written in terms of a single connective ‘notand’ where ‘ P notand Q ’ is interpreted as ‘not (P and Q)’.

To express ‘not P ’ we use $\boxed{\text{‘}P \text{ notand } P\text{’}}$.

To express ‘ P or Q ’ we note that it is equivalent to ‘not [(not P) and (not Q)]’. Therefore, it is equivalent to

‘not P ’ notand ‘not Q ’

and therefore to

$(P \text{ notand } P) \text{ notand } (Q \text{ notand } Q)$.

To express ‘ P and Q ’ we use the fact that it is equivalent to ‘not [(not P or (not Q)]’ and our previous work translating both ‘not’ and ‘and’ in terms of ‘notand’.