MATH 109 **Assignment 1**
Due: 4/4/11

#1.2(i)

P	Q	P and Q	not (P and Q)
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

(ii)

P	Q	not P	not Q	(not P) or (not Q)
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

(iii)

P	Q	not Q	P and (not Q)
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

(iv)

P	Q	not P	(not P) or Q
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

#2.5 (i)

P	Q	$P \Rightarrow Q$	not P	not Q	(not Q) $\Rightarrow$ (not P)
T	T	T	F	F	T
T	F	F	F	T	F
F	T	T	T	F	T
F	F	T	T	T	T

Since the third and last columns are the same, the statements are equivalent.

(ii)

P	Q	P or Q	not P	(not P) $\Rightarrow$ Q
T	T	T	F	T
T	F	T	F	T
F	T	T	T	T
F	F	F	T	F

Since the third and last columns are the same, the statements are equivalent.

#3.2 **Proof.** By definition, if (a divides b) and (b divides c) then there exist integers $x, y \in \mathbb{Z}$ such that $ax = b$ and $by = c$. Hence

$$a(xy) = (ax)y = by = c$$

so a divides c. $\square$

#3.6 **Proof.** By Axiom 3.1.2 (ii), if $a < b$ and b is negative then $b^2 < ab$. Similarly, $a < b$ and a is negative imply that $ab < a^2$. Hence $b^2 < ab < a^2$ as desired. $\square$

Part I Problems #1: For the solution to the first part of the problem, please see #2.5 (i). The contrapositive of the statement (i) ($f(a) = 0 \Rightarrow a > 0$) is (vii): For real numbers a, if a is nonpositive then $f(a) \neq 0$. The statements (iii) and (vi) are another pair of statements which are contrapositives of each other.