MATH 109	**Assignment 2**
Due: 4/11/11	

#4.2 **Proof.** We are given that n is odd, and we suppose for a contradiction that n^2 is not odd. By definition, this implies that $2 \nmid n$ and $2 \mid n^2$. Let $x \in \mathbb{Z}$ be the integer such that $2x = n^2$.

Observe that

$$(n+2)^2 = n^2 + 4n + 4 = 2x + 4n + 4 = 2(x + 2n + 2)$$

so $(n+2)^2$ is even. By Part I Problem #7, we have that $n+2$ must be even so let $y \in \mathbb{Z}$ be the integer such that $2y = n + 2$. Since

$$n = (n+2) - 2 = 2y - 2 = 2(y-1),$$

we get a contradiction to the fact that n is odd. Hence, n^2 must be odd. $\square$

#4.7 **Proof.** By Axiom 3.1.2 (ii), we can add the inequalities $-|a| \leq a \leq |a|$ and $-|b| \leq b \leq |b|$ to obtain

$$-(|a| + |b|) \leq a + b \leq |a| + |b|,$$

and hence we conclude that $|a + b| \leq |a| + |b|$. A necessary and sufficient condition for equality is that either

(i) ($a = 0$ or $b = 0$) or

(ii) $a = cb$ for some positive $c \in \mathbb{R}$. $\square$

#I.7 **Proof.** We are given that n^2 is even and we suppose, for a contradiction, that n is odd. This implies that $n = 2q + 1$ for some integer q. Hence

$$n^2 = (2q+1)^2 = 4q^2 + 4q + 1 = 2(2q^2 + 2q) + 1,$$

which contradicts the fact that n is even. Consequently, n must be even. $\square$

#6.2 (i) **Proof.** Suppose $x \in \mathbb{R}$ satisfies $0 = x^2 + x - 2 = (x-1)(x+2)$. By the table on page 37, we see that either $x - 1 = 0$ or $x + 2 = 0$. Hence $x = 1$ or $x = -2$. This proves that $\{x \in \mathbb{R} : x^2 + x - 2 = 0\} = \{1, -2\}$ as desired. $\square$

(ii) **Proof.** Suppose $x \in \mathbb{R}$ satisfies $0 > x^2 + x - 2 = (x-1)(x+2)$. By Proposition 4.4.3, either ($x - 1 > 0$ and $x + 2 < 0$) or ($x - 1 < 0$ and $x + 2 > 0$). Since there are no $x \in \mathbb{R}$ satisfying the first pair of conditions, we have that $x < 1$ and $x > -2$. This proves that $\{x \in \mathbb{R} : x^2 + x - 2 < 0\} = (-2, 1)$ as desired. $\square$

(iii) **Proof.** Suppose $x \in \mathbb{R}$ satisfies $0 < x^2 + x - 2 = (x-1)(x+2)$. By Proposition 4.4.2, either $(x - 1 > 0$ and $x + 2 > 0)$ or $(x - 1 < 0$ and $x + 2 < 0)$. Since $-2 < 1$, we see that $\{x \in \mathbb{R} : x - 1 > 0 \text{ and } x + 2 > 0\} = \{x \in \mathbb{R} : x > 1\}$. Similarly, we have $\{x \in \mathbb{R} : x - 1 < 0 \text{ and } x + 2 < 0\} = \{x \in \mathbb{R} : x < -2\}$. This proves that

$$\{x \in \mathbb{R} : x^2 + x - 2 > 0\} = \{x \in \mathbb{R} : x < -2\} \cup \{x \in \mathbb{R} : x > 1\},$$

as desired. □

#6.6 **Proof.** We are given that $A \cap B \subset C$ and $x \in B$. Suppose, for a contradiction, that $x \in A - C$. This implies that $x \in A$. As we are given that $x \in B$, we have

$$x \in A \cap B \subset C$$

so that $x \in C$. This contradicts our initial assumption that $x \in A - C$. Hence $x \notin A - C$. □