MATH 109	Assignment 8
Due: 6/1/11	

#22.3 **Proof.** To see that f is a surjection, suppose that $n \in \mathbb{Z}$. By trichotomy, we have $n > 0$, $n = 0$, or $n < 0$. If $n > 0$, then $n + 1 \in \mathbb{Z}^+$ and $f(n + 1, 1) = n$. If $n = 0$, then $f(1, 1) = 0 = n$. If $n < 0$, then $-n + 1 \in \mathbb{Z}^+$ and $f(1, -n + 1) = n$. This proves that f is a surjection. The equivalence classes of the relation defined by $(x_1, x_2) \sim (y_1, y_2) \Leftrightarrow f(x_1, x_2) = f(y_1, y_2)$ are of the form $\{(x_1, x_2) : x_1 - x_2 = k\}$ for some $k \in \mathbb{Z}$. □

#19.2 **Proof.** We are given that $m \mid (a_1 - a_2)$ and $m \mid (b_1 - b_2)$ so let $x, y \in \mathbb{Z}$ be the integers such that $a_1 - a_2 = mx$ and $b_1 - b_2 = my$ respectively. Since

$$(a_1 - b_1) - (a_2 - b_2) = (a_1 - a_2) + (b_1 - b_2)(-1) = mx - my = m(x - y),$$

we see that $m \mid (a_1 - b_1) - (a_2 - b_2)$ so $a_1 - b_1 \equiv a_2 - b_2 \pmod{m}$. □

#20.1 (i) **Solution:** To solve the congruence $3x \equiv 5 \pmod{11}$ we first solve the linear Diophantine equation $3x + 11y = 5$. We have

$$\begin{aligned} 11 &= 3(3) + 2 \\ 3 &= 2(1) + 1 \\ 2 &= 1(2) + 0. \end{aligned}$$

Rearranging these equations yields

$$1 = 3 - 2 = 3 - (11 - 3(3)) = 11(-1) + 3(4) \quad \Leftarrow 5 = 11(-5) + 3(20).$$

Hence $x = 20$ is a solution to $3x \equiv 5 \pmod{11}$. Since $20 \equiv 9 \pmod{11}$ and gcd(3,11)=1, we see that $x \equiv 9 \pmod{11}$ is the unique solution to $3x \equiv 5 \pmod{11}$ by Theorem 20.1.7. □

#21.3 Below is the multiplication table mod 12. We see that 5 is invertible and is its own inverse. Similarly, 7 is invertible and is its own inverse. Finally 11 is invertible and is its own inverse.

X	0	1	2	3	4	5	6	7	8	9	10	11
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11
2	0	2	4	6	8	10	0	2	4	6	8	10
3	0	3	6	9	0	3	6	9	0	3	6	9
4	0	4	8	0	4	8	0	4	8	0	4	8
5	0	5	10	3	8	1	6	11	4	9	2	7
6	0	6	0	6	0	6	0	6	0	6	0	6
7	0	7	2	9	4	11	6	1	8	3	10	5
8	0	8	4	0	8	4	0	8	4	0	8	4
9	0	9	6	3	0	9	6	3	0	9	6	3
10	0	10	8	6	4	2	0	10	8	6	4	2
11	0	11	10	9	8	7	6	5	4	3	2	1

#23.2 **Solution:** We have $4148 = 2^2 \cdot 17 \cdot 61$ and $7684 = 2^2 \cdot 17 \cdot 113$. Hence gcd(4148,7684) $= 2^2 * 17 = 68$. □