INSTRUCTIONS — READ THIS NOW

- This test has 5 problems on 1 page worth a total of 50 points.
- Write your name and PID on the cover of your Blue Book right now.
- Write your solutions clearly in your Blue Book. Carefully indicate the number and letter of each question and question part. Show your work. To receive full credit, your answers must be neatly written and logically organized. If you need more space, write on the back side of the preceding sheet, but be sure to label your work clearly. Partial credit will be awarded for demonstrating understanding of relevant definitions and theorem statements.
- No calculators are permitted but one handwritten page of notes may be used.
- This exam is 50 minutes long.
- Academic integrity is expected of all students at all times, whether in the presence or absence of members of the faculty.
1. (10 points) List all of the subgroups of the cyclic group $\mathbb{Z}_6$. Justify your answer. (Make sure your list has no duplicate subgroups).

2. (10 points) Consider

   \begin{align*}
   G_1 &= \mathbb{Z}_4 \quad \text{(under addition modulo 4),} \\
   G_2 &= U(5) \quad \text{(under multiplication modulo 5),} \\
   G_3 &= U(8) \quad \text{(under multiplication modulo 8).}
   \end{align*}

   For each pair of groups below, prove the groups are isomorphic or prove that they are not.

   (a) $G_1 \approx G_2$?
   (b) $G_1 \approx G_3$?

3. (10 points) Consider $U(9) = \{1, 2, 4, 5, 7, 8\}$ under multiplication modulo 9. Prove that $|\text{Inn}(U(9))| = 1$.

4. (10 points)

   (a) Let $G$ be the real numbers $\mathbb{R}$ under addition; let $H = \mathbb{Z}$, a subgroup of $G$. Describe the coset $\pi + H$.

   (b) How many distinct left cosets of

   \[ H = \{e, R_{36}, R_{72}, \ldots, R_{324}\} = \{R_x : x = 36n, 0 \leq n \leq 9\} \]

   are there in $G = D_{10}$? Justify your answer.

5. (10 points) Let $G$ be an abelian group and let $a, b \in G$ be elements satisfying

   \[ |a| = 3, \quad |b| = 3, \quad a \notin \langle b \rangle, \quad b \notin \langle a \rangle. \]

   Prove that $|G|$ is divisible by 9.

   **Hint:** show that the set \{e, a, a^2, b, ab, a^2b, b^2, ab^2, a^2b^2\} is a subgroup of $G$. 