Consider the code \( f : \{0, 1\}^3 \to \{0, 1\}^6 \) given by the generator matrix
\[
\begin{pmatrix}
1 & 0 & 0 & 1 & 1 & 0 \\
0 & 1 & 0 & 0 & 1 & 1 \\
0 & 0 & 1 & 1 & 1 & 1 \\
\end{pmatrix}
\]

(1) Let \( H \) be the set of codewords,
\[
H = \{ f(x) : x \in \{0, 1\}^3 \}.
\]
List the elements of \( H \).

(2) List the elements of the coset
\[
011001 + H.
\]

(3) How many distinct (left) cosets of \( H \) are there in \( G = \{0, 1\}^6 \)?
(1) 
\[ H = \{ f(x) : x \in \{0, 1\}^3 \} = \{ f(000), f(001), f(010), f(011), f(100), f(101), f(110), f(111) \} \]
\[ = \{000000, 001111, 010011, 011100, 100110, 101001, 110101, 111010\} \]

(2) 
\[ 011001 + H = \{011001, 010110, 001010, 000101, 111111, 110000, 101100, 100011\} \]

(3) By Lagrange's theorem, \[ |G : H| = \frac{|G|}{|H|} = \frac{2^6}{8} = 8. \]