**MATH 103A Homework 3**

**DUE January 25, 2013**

**Version January 19, 2013**

**Assigned reading:** Chapters 1-3 of Gallian.

**Recommended practice questions:** Chapter 1 of Gallian, exercises

1, 2, 4, 5, 6, 7, 11

Chapter 2 of Gallian, exercises

17, 31, 33, 34, 44, 46, 47, 48, 49

Chapter 3 of Gallian, exercises

1, 7

**Assigned questions to hand in:**

1. *(Gallian Chapter 1 # 13)* Describe the symmetries of a nonsquare rectangle. Construct the corresponding Cayley (multiplication) table.

2. *(Gallian Chapter 2 # 18)* List the members of $H = \{ x^2 : x \in D_4 \}$ and $K = \{ x \in D_4 : x^2 = e \}$.

3. *(Gallian Chapter 2 # 32)* Construct a Cayley (multiplication) table for $U(12)$.

4. *(Gallian Chapter 2 # 36)* Let $a, b$ belong to a group $G$. Find an $x$ in $G$ such that $xabx^{-1} = ba$.

5. *(Gallian Chapter 3 # 4)* Prove that in any group, an element and its inverse have the same order.

6. *(Gallian Chapter 3 # 6)* In the group $Z_{12}$, find $|a|$, $|b|$, and $|a + b|$ for each case.
   
   (a) $a = 6$, $b = 2$
   
   (b) $a = 3$, $b = 8$
   
   (c) $a = 5$, $b = 4$
   
   Do you see any relationship between $|a|$, $|b|$, and $|a + b|$?

7. *(Gallian Chapter 3 # 32)* If $H$ and $K$ are subgroups of $G$, show that $H \cap K$ is a subgroup of $G$. (Can you see that the same proof shows that the intersection of any number of subgroups of $G$, finite or infinite, is again a subgroup of $G$?)