Let \( g(t) \) be grams in tank at time \( t \), noting \( g(0) = 200 \text{g} \).

Notice the volume \( (\text{liters}) \) of the tank is constant.

Next, \[
\frac{dg}{dt} = \text{rate in} - \text{rate out}
\]

\[
= \left( \frac{2 \text{ liters}}{\text{min}} \cdot \frac{0 \text{ grams}}{\text{liter}} \right) - \left( \frac{2 \text{ liters}}{\text{min}} \cdot \frac{g(t) \text{ grams}}{200 \text{ liters}} \right)
\]

\[
= \frac{0 \text{ grams}}{\text{liter}} - \frac{g(t) \text{ grams}}{100 \text{ liter}},
\]

So our eq'n is \( g' = \frac{-g}{100} \), or \( g' + \frac{g}{100} = 0 \)

Now use integrating factors to solve.

Our I.F. is \( e^{\int \frac{1}{100} dt} \), so we have

\[
\frac{d}{dt} \left( g e^{\frac{t}{100}} \right) = 0 \cdot e^{\frac{t}{100}} = 0
\]

So \( g e^{\frac{t}{100}} = c \) \( \implies g = c e^{-\frac{t}{100}} \).

Now use \( g(0) = 200 \) to get \( g(t) = 200 e^{-\frac{t}{100}} \).

Lastly, 1\% of \( g(0) = (0.01)(200) = 2 \text{ grams} \), so we solve \( 2 = 200 e^{-\frac{t}{100}} \) for \( t \), so

\[
t = 100 \log(100)
\]
Let $g(t)$ be the number of grams of salt in the tank at time $t$. $g(0) = 0$.

Then \[
\frac{dg}{dt} = \text{(rate in) - (rate out)}
\]
\[
= \left( \frac{2 \text{ liters}}{\min} \cdot \frac{1 \text{ gram}}{\text{liter}} \right) - \left( \frac{2 \text{ liters}}{\min} \cdot \frac{g(t) \text{ grams}}{120 \text{ liters}} \right)
\]
\[
= (2 \gamma - \frac{g(t)}{60}) \frac{\text{grams}}{\text{liters}}
\]

So we have \[
\frac{g'}{60} + \frac{g}{60} = 2 \gamma^6. \quad \text{Our integrating factor is}
\]
\[
e^{\frac{t}{60}} \quad \text{so we have} \quad \frac{d}{dt}(ge^{\frac{t}{60}}) = 2 \gamma e^{\frac{t}{60}}. \quad \text{Integrate}
\]
\[
ge^{\frac{t}{60}} = 120 \gamma e^{\frac{t}{60}} + c
\]
\[
\text{or} \quad g(t) = 120 \gamma + c e^{-\frac{t}{60}}
\]
\[
0 = g(0) = 120 \gamma + c \quad \text{so} \quad c = -120 \gamma \quad \gamma
\]
\[
g(t) = -120 \gamma e^{-\frac{t}{60}} + 120 \gamma
\]

\[
\lim_{t \to \infty} g(t) = 120 \gamma
\]
&
Put \( p(t) \) = pounds (lbs) of salt in our tank at time \( t \),
\( p(0) = 0 \).

Now \( \frac{dp}{dt} = \text{rate in} - \text{rate out} \)
\[ = \left( \frac{2 \text{ gal}}{\text{min}} \cdot \frac{1 \text{ lb}}{2 \text{ gal}} \right) - \left( \frac{2 \text{ gal}}{\text{min}} \cdot \frac{p(t) \text{ lbs}}{100 \text{ gal}} \right) \]
\[ = \left( 1 - \frac{p(t)}{50} \right) \frac{\text{lbs}}{\text{gal}} \text{ for the first 10 minutes.} \]

So \( p'(t) = 1 - \frac{p}{50} \) or \( p' + \frac{p}{50} = 1 \). An I.F. is \( e^{\frac{t}{50}} \), so
\[ \frac{d}{dt} \left( p e^{\frac{t}{50}} \right) = e^{\frac{t}{50}} \text{ & integrating gives} \]
\[ pe^{\frac{t}{50}} = 50e^{\frac{t}{50}} + C \implies p(t) = 50 + Ce^{-\frac{t}{50}} \]
\[ 0 = p(0) = 50 + C \implies C = -50. \]
\( \Rightarrow p(t) = 50 - 50e^{-\frac{t}{50}} \) for the first 10 minutes.

Now for the 2nd 10 minutes, \( \frac{dp}{dt} = \left( \frac{2 \text{ gal}}{\text{min}} \cdot 0 \frac{\text{lbs}}{\text{gal}} \right) - \left( \frac{2 \text{ gal}}{\text{min}} \cdot \frac{p(t) \text{ lbs}}{100 \text{ gal}} \right) \)
\[ = -\frac{p(t)}{50} \frac{\text{lbs}}{\text{gal}} \]

So \( p' + \frac{p}{50} = 0 \) & w/ I.F. \( e^{\frac{t}{50}} \), we see \( \frac{d}{dt} \left( p e^{\frac{t}{50}} \right) = 0 \).

\( \Rightarrow p e^{\frac{t}{50}} = C \text{ or } p = Ce^{-\frac{t}{50}} \). Now \( 50(1 - e^{-\frac{t}{50}}) = p(10) = e^{-\frac{1}{5}} \),
\[ \Rightarrow p(20) = 50(e^{\frac{1}{5}}) e^{-\frac{2}{5}} \times 7.42 \text{ lbs} \]
2.3.12. $Q(t)$ - amount of carbon at time $t$.
$Q_0 = Q(0)$.

(a) $Q' = -rQ$

$\Rightarrow Q' + rQ = 0$, use int. factor $e^{rt}$, get

$$\frac{d}{dt}(Qe^{rt}) = 0 \Rightarrow Qe^{rt} = C \Rightarrow Q = ce^{rt}.$$ 

At $t = 0$ we have $Q_0 = Q(0) = C$, so $Q(t) = Q_0 e^{rt}$.

Next since the half life is 5730 yrs,

$$\frac{Q_0}{2} = Q(5730) = Q_0 e^{5730r}.$$

Assuming $Q_0 \neq 0$ we have

$$\frac{1}{2} = e^{5730r} \Rightarrow \frac{\log(\frac{1}{2})}{5730} = r$$

(or $r \approx -0.00012097\text{yr}^{-1}$).

(b) We solved (b) in (a), i.e $Q(t) = Q_0 e^{rt}$ will use above.

(c) At time $t = 0$, $Q_0 = Q(0)$.

Let $\tilde{t}$ be # of years from time $t = 0$ until today.

Then $0.2Q_0 = Q(\tilde{t}) = Q_0 e^{rt}$

given

Here $0.2 = e^{r\tilde{t}}$, so $\tilde{t} = \frac{\log(0.2)}{r} = \frac{\log(0.2) \times 5730 \text{ years}}{\log(0.5)} \approx 13304.698 \text{ yrs}$.
Let \( x(t) \) be the ball's height, \( x' = v \) the ball's velocity.

Notice \( x(0) = 30 \text{m} \) \( v(0) = 20 \text{m/s} \).

Now, \( 15v' = \text{Total Force} = -15g \)

\[ v' = -9.8 \quad \text{or} \quad v = -9.8t + c \]

So \( 20 = v(0) = 0 \) \& we have \( v(t) = -9.8t + 20 \)

Next, \( x' = v = -9.8t + 20 \), so \( x(t) = -\frac{9.8}{2}t^2 + 20t + c \).

Now \( 30 = x(0) = c \), so \( x(t) = -\frac{9.8}{2}t^2 + 20t + 30 \).

(a) To find \( x \)'s maximum set \( x' = v = 0 \), so \( t = \frac{20}{9.8} \approx 2.04 \)

At this time \( x\left(\frac{20}{9.8}\right) \approx 50.41 \text{m} \).

(b) \( 0 = x(t) = -\frac{9.8}{2}t^2 + 20t + 30 \). Use quad. formula,

\[ t = \frac{-20 \pm \sqrt{400 + 120 \cdot 4.9}}{-9.8} \approx -1.16 \text{ or } 5.25 \]

Choose the positive one \( t \approx 5.25 \).

(c) \( x(t) \) vs. \( t \) and \( v(t) \) vs. \( t \) graphs.
We are given an air resistance force of \( \frac{V^2}{1365} = \frac{V^2}{a} \) say.

So our total force is \( \frac{mgV^2}{a} = -mgV = \frac{-mgV^2}{a} \).

With mass \( m = 0.15 \text{ kg} \), \( g = 9.8 \), \( V(t) \) velocity.

Separate variables:

\[
\int \frac{dv}{-mg - V^2} = \int \frac{dt}{ma} = \frac{t}{ma} + c
\]

\[
\frac{1}{-mg} \int \frac{dv}{1 + \left(\frac{v}{u}\right)^2} \quad \text{with} \quad u = \left(\frac{1}{mg}\right)^{\frac{1}{2}}
\]

Let \( s = \frac{V}{u} \), so \( ds = \frac{dv}{u} \), & we have:

\[
\frac{u}{-mg} \int \frac{ds}{1 + s^2} = \frac{u}{-mg} \arctan(s) = \frac{u}{-mg} \arctan\left(\frac{v}{u}\right).
\]

We have shown that

\[
\frac{u}{-mg} \arctan\left(\frac{v}{u}\right) = \frac{t}{ma} + c, \quad \text{so} \quad V = \tan\left(\frac{1}{kmag} + c\right) \times u, \quad \text{here} \quad c = \frac{u}{kmag}
\]

Now use \( V(0) = 20 \) to find \( c' \). Let \( c' = \frac{1}{kmag} \).

Then \( x = \int V \, dt = \mu \int \frac{\sin{(at + c')}}{\cos{(at + c')}} \, dt \), \( \text{Put} \quad s = \cos{(at + c')} \)

\[
= \frac{\mu}{\alpha} \int \frac{ds}{\cos s} = \frac{\mu}{\alpha} \log |\cos(at + c')| + c.
\]

Use \( x(0) = 30 \) to find \( c \), & we have solved the initial value problem.

Then answer the questions.

Overall be careful with the constants here.
\[
\frac{y' + 2ty}{4-t^2} = \frac{3t^2}{4-t^2}, \quad y(-3) = 1.
\]

So \[p(t) = \frac{2t}{4-t^2} \quad \text{and} \quad q(t) = \frac{3}{4-t^2} \]
p and \(q\) are defined and continuous whenever \(-4-t^2 \neq 0\)
or \(t^2 > 4\), or \(t \in (-\infty, -2) \cup (-2, 2) \cup (2, \infty)\).

Now our initial data is \(y(-3) = 1\), so \(t = -3\) lies in \((-\infty, -2)\).

Any interval containing \(-3\) and contained in \((-\infty, -2)\)
will suffice.

\[
y' = (1-t^2-y^2)^{1/2} = f(t,y).
\]

\(f\) is defined and continuous whenever \(1-t^2-y^2 > 0\)
or \(t^2-y^2 < 1\), which is defined and continuous on the same set. Graphically it is the open unit disk.

So for any initial point \(y(t_0) = y_0, w(t_0, y_0)\) in the disk, we can apply Theorem 2.4.2.
\[ y' = \frac{-t + (t^2 + 4y)^{1/2}}{2}, \quad y(2) = -1. \]

So using notation from Thm. 2.4.2,

\[ f(t, y) = \frac{-t}{2} + \frac{1}{2} (t^2 + 4y)^{1/2}. \] It is continuous and defined whenever \( t + 4y \) satisfies \( t^2 + 4y > 0 \) or \( t^2 > -4y \).

\[ \frac{df}{dy} = (t^2 + 4y)^{-1/2}, \text{ defined & continuous whenever } \]

\[ t^2 + 4y > 0 \text{ or } t^2 > -4y. \] (the same region)

Graphically, \( y \sim \), & the curve is not included in the region (only the shaded area under the curve).

Now to apply 2.4.2, we need a rectangle \( a < t < b, \quad f \)

\( a < y < b \) containing our initial condition point \((2, -1)\),

such that the rectangle is in our region where \( f \) & \( \frac{df}{dy} \) are defined & continuous. But \((2, -1)\) lies on the curve, hence is not inside the region, & so clearly any rectangle containing \((2, -1)\) is not contained in the region. So Thm. 2.4.2 does not apply.