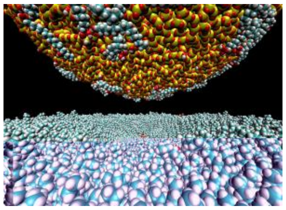
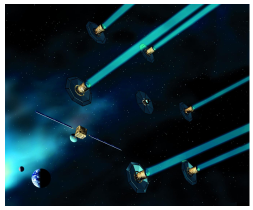
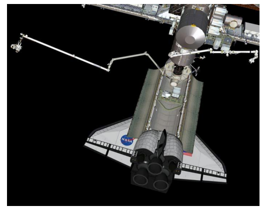
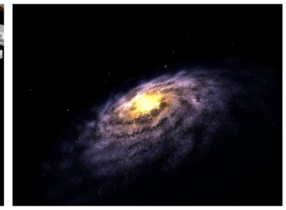
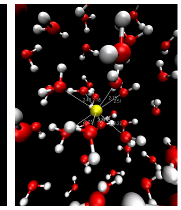
MATH 692A: Geometric Numerical Integration

TTh 12-1:15pm, MATH 211

Instructor

Prof. Melvin Leok, Department of Mathematics
Office: Math 430, phone: 49-63578, email: `mleok@math.purdue.edu`
`http://www.math.purdue.edu/~mleok/`

Office Hours

Tuesday and Thursday, 2:30pm-3:30pm. You may also attend my TTh 3:30pm-4:30pm office hours, but priority will be given to my MATH 366 students.

URLs

Course Website: `http://www.math.purdue.edu/~mleok/courses/ma692a.html`
Course Announcements: `http://intranet.math.purdue.edu/ma692aspring2008/`

Target Audience

This course is relevant to engineers, scientists, and mathematicians with an interest in long-time simulations of mechanical systems, including applications to robotic motion planning, astrodynamics, rigid-body, molecular and stellar dynamics. The application areas addressed will be tailored to the interests of the course participants.

Course Description

Many differential equations of interest in the physical sciences and engineering exhibit geometric properties that are preserved by the dynamics. Recently, there has been a trend towards the construction of numerical schemes that preserve as many of these geometric invariants as possible.

Such methods are of particular interest when simulating mechanical systems that arise from Lagrangian or Hamiltonian mechanics, wherein the preservation of physical invariants such as the energy, momentum, and symplectic form can be important when simulating long-time dynamics of such systems.

In applications arising from astrodynamics and robotics, the dynamics evolve on nonlinear manifolds such as Lie groups, and in particular the rotation group, and the special Euclidean group. Numerical schemes that respect the underlying nonlinear manifold structure will also be discussed.

This course will begin with an overview of classical numerical integration schemes, and their analysis, followed by a more in depth discussion of the various geometric properties that are of importance in many practical applications, followed by a survey of the various geometric integration schemes that have been developed in recent years. Issues pertaining to the analysis and implementation of such schemes will also be addressed.

Background

A strong undergraduate background in linear algebra, and differential equations; Some familiarity with numerical methods, classical mechanics and differential geometry would be helpful, but not essential; Programming experience in any language, e.g., C/C++, FORTRAN, MATLAB.

Textbook

Simulating Hamiltonian Dynamics
Cambridge Monographs on Applied and Computational Mathematics),
Leimkuhler, Reich, Cambridge University Press, 2005. ISBN: 0521772907

Additional Reading

Geometric Numerical Integration
(Springer Series in Computational Mathematics),
Hairer, Lubich, Wanner, 2nd Edition, Springer-Verlag 2006. ISBN: 3540306633

Course Topics Numerical methods, Hamiltonian mechanics, Geometric integrators, Modified equations, Higher-order methods, Constrained mechanical systems, Rigid body dynamics, Adaptive geometric integrators, Highly oscillatory problems, Molecular dynamics, Hamiltonian PDEs.

Grading

40 % – 4 Theoretical homework assignments

20 % – 2 Numerical implementation projects

30 % – Final project involving a non-trivial application of geometric integrators.

5 % – Final presentation

5 % – Class participation

Collaboration Policy Homework is an essential part of advanced mathematics courses. Most students will find that some problems will require repeated and persistent effort to solve. This process is an integral component of developing a mastery of the material presented, and students who do not dedicate the necessary time and effort towards this will compromise their understand of the material in this course, and their ability to apply this material in their subsequent work.

A student may after working conscientiously on a problem for over 30 minutes, consult with other current Math 692A students to develop and clarify their approach to the problem. The written solution should however be an independent and individual effort that reflects the student's understanding of the problem and its solution.

As a general guide, a student should be able to independently reproduce any solution that is submitted as homework. Copying of solutions is not permitted and will be considered a violation of these guidelines.