MATH 170B ASSIGNMENT 3

§6.1, 9: Prove that if g interpolates the function f at $x_0, \ldots, x_{n-1}$ and if h interpolates f at $x_1, \ldots, x_n$, then the function

$$g(x) + \frac{x_0 - x}{x_n - x_0}[g(x) - h(x)]$$

interpolates f at $x_0, \ldots, x_n$. Notice that g and h need not be polynomials.

§6.1, 14: Let p be a polynomial of degree $\leq n-1$ that interpolates the function $f(x) = \sinh x$ at any set of n nodes in the interval $[-1, 1]$, subject only to the condition that one of the nodes is 0. Prove that the error obeys this inequality on $[-1, 1]$:

$$|p(x) - f(x)| \leq \frac{2^n}{n!}|f(x)|$$

§6.1, 22: Find the Lagrange and Newton forms of the interpolating polynomial for these data:

x	-2	0	1
$f(x)$	0	1	-1

Write both polynomials in the form $a+bx+cx^2$ to verify that they are identical as functions.

§6.2, 8: Using the functions l_i defined in Section 6.1 (p.312) and based on nodes $x_0, \ldots, x_n$, show that for any f,

$$\sum_{i=0}^{n} f(x_i)l_i(x) = \sum_{i=0}^{n} f[x_0, \ldots, x_i] \prod_{j=0}^{i-1}(x - x_j).$$

§6.2, 9: (Continuation) Prove this formula:

$$f[x_0, \ldots, x_n] = \sum_{i=0}^{n} f(x_i) \prod_{j=0, j\neq i}^{n} (x_i - x_j)^{-1}$$

§6.2, 24: Use divided differences to write the Newton interpolating polynomial for these data:

x	4	2	0	3
$f(x)$	63	11	7	28