MATH 170B HOMEWORK 3 SOLUTIONS

§6.1: Questions 9, 14, 22

9. Let $g(x_i) = f(x_i)$ for $0 \le i \le n-1$ and $h(x_i) = f(x_i)$ for $1 \le i \le n$. Set $k(x) = g(x) + [(x_0 - x)/(x_n - x_0)][g(x) - h(x)]$. Then $k(x_0) = g(x_0) = f(x_0)$ and for $1 \le i \le n-1$ we have $k(x_i) = g(x_i) + [(x_0 - x_i)/(x_n - x_0)][g(x_i) - h(x_i)] = g(x_i) = f(x_i)$ and $k(x_n) = h(x_n) = f(x_n)$.

14. Say $x_j = 0$. $|p(x) - f(x)| = \left|(1/n!)f^{(n)}(\xi_x)\prod_{i=0}^{n-1}(x - x_i)\right|$

$\le (1.5431/n!)|x|\prod_{\substack{i=0\\ i\neq j}}^{n-1}(x - x_i) \le (1.5431/n!)|x|2^{n-1}$ since node x_j is 0. Note as in Problem 6.1.13,

$f^{(n)}(\sinh x) = \begin{cases} \sinh x & n \text{ even} \\ \cosh x & n \text{ odd} \end{cases}$

and $|f^{(n)}(\sinh x)| \le \max\{\sinh 1, \cosh 1\}$, on $[-1, 1]$.

So $|p(x) - f(x)|/|f(x)| \le (1.5431/n!)|x/\sinh x|2^{n-1} \le (1.5431/n!)2^{n-1} \le (2^n/n!)$

since $x/\sinh x \le 1$.

22. Lagrange form: $p(x) = -(1/2)(x+2)(x-1) - (1/3)x(x+2) = -(1/6)(5x^2 + 7x - 6)$.

Newton form:

x	$f(x)$		
-2	0	$1/2$	$-5/6$
0	1	-2	
1	-1		

$p(x) = (1/2)(x+2) - (5/6)(x+2)x = -(1/6)(5x^2 + 7x - 6)$.

§6.2: Question 8, 9, 24

8. LHS is the Lagrange interpolating polynomial of degree $\le n$ for f at nodes $x_0, x_1, \ldots, x_n$. RHS is the Newton interpolating polynomial of degree $\le n$ for f at $x_0, x_1, \ldots, x_n$. Hence, LHS equals RHS by uniqueness.

9. By Problem 6.2.8, the two polynomials are equal and hence the coefficients of x^n in each are equal.

24.

x	$f(x)$			
4	63	26	6	1
2	11	2	5	
0	7	7		
3	28			

Thus, $p(x) = 63 + 26(x-4) + 6(x-4)(x-2) + x(x-4)(x-2)$.