MATH 170B ASSIGNMENT 4

§6.4, 5: Determine whether this is a quadratic spline function:

$$f(x) = \begin{cases} x & x \in (-\infty, 1] \\ -\frac{1}{2}(2-x)^2 + \frac{3}{2} & x \in [1, 2] \\ \frac{3}{2} & x \in [2, \infty) \end{cases}$$

Recall that a quadratic spline function is a piecewise quadratic polynomial, which is continuous and has continuous first derivatives at the knot points.

§6.4, 7: Determine all the values of a, b, c, d, e, for which the following function is a cubic spline:

$$f(x) = \begin{cases} a(x-2)^2 + b(x-1)^3 & x \in (-\infty, 1] \\ c(x-2)^2 & x \in [1, 3] \\ d(x-2)^2 + e(x-3)^3 & x \in [3, \infty) \end{cases}$$

§6.4, 9: Using the development of cubic splines as a model, derive the appropriate equations and algorithm to provide a quadratic spline interpolant to data (t_i, y_i) for $0 \le i \le n$, where $t_0 < t_1 < \cdots < t_n$. If Q is the spline interpolant, then the numbers $z_i = Q'(t_i)$ are well-defined. Find the equations governing $z_0, z_1, \ldots, z_n$. You should discover that one of the z points can be arbitrary, say $z_0 = 0$.

Taken from Burden & Faires, 5th edition, 1993: A natural cubic spline S on $[0, 2]$ is defined by

$$S(x) = \begin{cases} S_0(x) = 1 + 2x - x^3 & \text{if } 0 \le x < 1 \\ S_1(x) = 2 + b(x-1) + c(x-1)^2 + d(x-1)^3 & \text{if } 1 \le x \le 2\,. \end{cases}$$

Find b, c, and d.

Taken from Burden & Faires, 5th edition, 1993: A clamped cubic spline S for a function $f(x)$ on $[1, 3]$ is defined by

$$S(x) = \begin{cases} S_0(x) = 3(x-1) + 2(x-1)^2 - (x-1)^3 & \text{if } 1 \le x < 2 \\ S_1(x) = a + b(x-2) + c(x-2)^2 + d(x-2)^3 & \text{if } 2 \le x \le 3\,. \end{cases}$$

Given that $f'(1) = f'(3)$, find a, b, c, and d.