MATH 170B ASSIGNMENT 5

§6.5, 1: Verify this formula for $B_j^2(t_i)$,

$$B_j^2(t_i) = \left(\frac{t_i - t_{i-1}}{t_{i+1} - t_{i-1}}\right)\delta_{i,j+1} + \left(\frac{t_{i+1} - t_i}{t_{i+1} - t_{i-1}}\right)\delta_{i,j+2}$$

§6.5, 3: Let $h_i = t_{i+1} - t_i$. Show that if

$$c_{i-1}h_{i-1} + c_{i-2}h_i = y_i(h_i + h_{i-1})$$

then the spline function $S = \sum_{i=-\infty}^{\infty} c_i B_i^2$ will have the interpolatory property $S(t_j) = y_j$ for all j.

§6.5, 7: Prove that

$$\int_{-\infty}^{\infty} B_i^k(x)dx = \frac{t_{i+k+1} - t_i}{k+1}$$

§6.6, 8: Prove that if $S = \sum_{j=-\infty}^{\infty} c_j B_j^3$, then $S'' = \sum_{j=-\infty}^{\infty} e_j B_j^1$ with

$$e_j = \frac{6}{t_{j+2} - t_j}\left(\frac{c_j - c_{j-1}}{t_{j+3} - t_j} - \frac{c_{j-1} - c_{j-2}}{t_{j+2} - t_{j-1}}\right)$$

§6.6, 9: (Continuation) Prove that if $S = \sum_{j=-\infty}^{\infty} c_j B_j^3$, then $S''(t_i) = e_{i-1}$, where e_j is defined in the previous problem.

§6.5, 2 (Computer problem): Let knots $t_1, t_2, \ldots, t_{n+k+1}$ be specified, as well as n and k. Let coefficients $c_1, c_2, \ldots, c_n$ be given, and write $f(x) = \sum_{i=1}^{n} c_i B_i^k(x)$. Write a subroutine that gives the value of $f(x)$ for any real value of x. Hint: Review the lemma on the recurrence relation for B-splines, and the associated numerical procedure and algorithm.