MATH 170B HOMEWORK 5 SOLUTIONS

§6.5: Questions 1, 3, 7

1. By Lemma 1, $B_i^2(x) = 0$ on the complement of (t_i, t_{i+3}). Use Equation (1) with $k = 2$ to compute $B_i^2(t_{i+1}) = (t_{i+1} - t_i)/(t_{i+2} - t_i)$ and $B_i^2(t_{i+2}) = (t_{i+3} - t_{i+2})/(t_{i+3} - t_{i+1})$. All other $B_i^2(t_j)$ are zero.
3. $S(t_m) = \sum_i c_i B_i^2(t_m) = c_{m-2}B_{m-2}^2(t_m) + c_{m-1}B_{m-1}^2(t_m)$
$= c_{m-2}h_m/(h_m + h_{m-1}) + c_{m-1}h_{m-1}/(h_m + h_{m-1}) = (c_{m-2}h_m + c_{m-1}h_{m-1})/(h_m + h_{m-1}) = y_m$.
7. Just let x tend to ∞ in Lemma 7, and use Lemma 4.

§6.6: Questions 8, 9

8. $S = \sum_{j=-\infty}^{\infty} c_j B_j^3$.
Theorem: If $f(x) = \sum_{i=-\infty}^{\infty} c_i B_i^k(x)$, then $f'(x) = \sum_{i=-\infty}^{\infty} d_i B_i^{k-1}(x)$
where $d_i = k(c_i - c_{i-1})/(t_{i+k} - t_i)$.
Now, $S' = \sum_{j=-\infty}^{\infty} d_j B_j^2$ where $d_j = 3(c_j - c_{j-1})/(t_{j+3} - t_j)$
and $S'' = \sum_{j=-\infty}^{\infty} e_j B_j^1$ where $e_j = 2(d_j - d_{j-1})/(t_{j+2} - t_j)$, $e_j = [2/(t_{j+2} - t_j)][d_j - d_{j-1}]$
$= [6/(t_{j+2} - t_j)][(c_j - c_{j-1})/(t_{j+3} - t_j) - (c_j - c_{j-2})/(t_{j+2} - t_{j-1})]$.
9. $S''(t_i) = \sum_{j=-\infty}^{\infty} e_j B_j^1(t_i) = e_{i-1}B_{i-1}^1(t_i) = e_{i-1}$ (using Problem 6.6.8 and definition of B_{i-1}^1).