MATH 170B ASSIGNMENT 6

§6.8, 3: Suppose that we wish to approximate an even function by a polynomial of degree $\leq n$ using the norm $\|f\| = \left\{\int_{-1}^{1} |f(x)|^2 dx\right\}^{1/2}$. Prove that the best approximation is also even. Hint: If $g(x)$ is the best approximation, look at how $h(x) = g(-x)$ approximates the function $f(x)$ (which is even), and use the uniqueness property of the best approximation in an inner-product space.

§6.8, 5: Prove the Parseval identity:

$$\langle f, g\rangle = \sum_{i=1}^{n} \langle f, u_i\rangle\langle g, u_i\rangle$$

which is valid if f and g are in the span of the orthonormal set $[u_1, u_2, \ldots, u_n]$.

§6.8, 9: Let $\{v_1, v_2, \ldots, v_n\}$ be an orthogonal (not necessarily orthonormal) set of vectors in an inner-product space. What choice of coefficients produces a minimum value in $\|f - \sum_{i=1}^{n} c_i v_i\|$? Don't overlook the possibility that some of the v's may be 0.

§6.8, 20: Find a formula for $\text{dist}(f, G)$, where G is the subspace spanned by an orthonormal set $[g_1, g_2, \ldots, g_n]$. Hint: Recall that by definition, if $g \in G$ is the best approximation of f in G, then $\text{dist}(f, G) = \text{dist}(f, g)$.

§6.8, 23: Determine p_3 in the form $p_3 = x^3 + Bx^2 + cx + D$ by making p_3 orthogonal to the space of polynomials of degree ≤ 2.