

MATH 170B: COMPUTER PROJECT #2  
 “POLYNOMIAL INTERPOLATION”  
 WINTER 2011

**NOTE:** For each homework assignment observe the following guidelines:

- Always clearly label all plots (title,  $x$ -label,  $y$ -label, and legend).
- Use the `subplot` command from MATLAB when comparing 2 or more plots to make comparisons easier and to save paper.

1. **SOURCE CODE:**

Let  $x_0, x_1, \dots, x_n$  be distinct points with corresponding data values  $f_0, f_1, \dots, f_n$ . Let  $P(x)$  be the unique polynomial of degree  $n$  which interpolates the data (i.e.,  $P(x_i) = f_i$  for each  $i = 0, \dots, n$ ). Write the following MATLAB routines:

- (a) `F = DivDiff(x,f,n)` – a **function** to compute the divided differences.
- (b) `P = Horners(x,F,xbar)` – a **function** which uses the divided differences to evaluate  $P(x)$  at the point `xbar` using Horner’s method.

2. Consider the function

$$f(x) = \frac{1}{1 + 25x^2}.$$

- (a) Let  $x_i$  be 11 equally spaced points on  $[-1, 1]$ :

$$x_i = -1 + i/5, \quad \text{where } i = 0, 1, 2, \dots, 10.$$

- i. Use your code from the previous problem to construct the divided differences with these  $x_i$ ’s.
- ii. Use your Horner’s method code to evaluate the interpolating polynomial  $P(x)$  at all the points  $z_k$ :

```
z = linspace(-1,1,500);
for k=1:500
    P(k) = Horners(x,F,z(k));
end
```

- iii. Make a single MATLAB plot that contains:  $(z, P)$ ,  $(z, f(z))$ , and the data points  $(x_i, f(x_i))$  for  $i = 0, 1, 2, \dots, 10$ . Make the  $P(z)$  plot a solid line (`'b-'`), the  $f(z)$  a dashed line (`'r--'`), and the data points stars (`'k*'`).

- (b) Now, let  $x_i$  be 11 *unequally* spaced points on  $[-1, 1]$ :

$$x_i = \cos \left[ \frac{\pi (i + 0.5)}{11} \right], \quad \text{where } i = 0, 1, 2, \dots, 10.$$

Repeat steps (i), (ii), and (iii) for these unequally spaced data points.

- (c) Explain what you observe with these two choices of the  $x_i$ ’s. Hint: Consider the form of the remainder term in polynomial interpolation.