MATH 170B ASSIGNMENT 1

§1.3, 10: Develop a complete theory for the difference equation $E^r x = 0$.

§1.3, 11: Give bases consisting of real sequences for each solution space.

a: $(4E^0 - 3E^2 + E^3)x = 0$

c: $(2E^6 - 9E^5 + 12E^4 - 4E^3)x = 0$

§1.3, 13: Solve

a: $x_{n+1} - nx_n = 0$

b: $x_{n+1} - x_n = n$

c: $x_{n+1} - x_n = 2$

§1.3, 14: Define an operate Δ by putting

$$\Delta x = [x_2 - x_1, x_3 - x_2, x_4 - x_3, \ldots]$$

Show that $E = 1 + \Delta$. Show that if p is a polynomial, then

$$p(E) = p(I) + p'(I)\Delta + \frac{1}{2}p''(I)\Delta^2 + \frac{1}{3!}p'''(I)\Delta^3 + \cdots + \frac{1}{m!}p^{(m)}(I)\Delta^m$$

§1.3, 27: Consider the recurrence relation $x_n = 2(x_{n-1} + x_{n-2})$. Show that the general solution is $z_n = \alpha(1+\sqrt{3})^n + \beta(1-\sqrt{3})^n$. Show that the solution with starting values $x_1 = 1$ and $x_2 = 1 - \sqrt{3}$ corresponds to $\alpha = 0$ and $\beta = (1-\sqrt{3})^{-1}$.