MATH 170B ASSIGNMENT 2

§3.1, 4: Derive the formula

$$n \geq \frac{\log(b_0 - a_0) - \log \epsilon}{\log 2} - 1$$

involving $b_0 - a_0$ and ϵ for the number of steps n that must be taken in the bisection method to guarantee that $|\alpha - \alpha_n| \leq \epsilon$.

§3.1, 5: Derive the formula

$$n \geq \frac{\log(b_0 - a_0) - \log \epsilon - \log a_0}{\log 2} - 1$$

involving a_0, b_0, and ϵ for the number of steps n that should be taken in the bisection algorithm to ensure that the root is determined with *relative* accuracy $\leq \epsilon$. Assume $a_0 > 0$.

§3.1, 7: If the bisection method is used starting with the interval $[2, 3]$, how many steps must be taken to compute a root with absolute accuracy $< 10^{-6}$? Answer the same question for the *relative* accuacy. What about to full precision on the `Marc-32` in each case?

§3.2, 5: What is the purpose of the following iteration formula?

$$x_{n+1} = 2x_n - x_n^2 y$$

Identify it as the Newton iteration for a certain function.

§3.2, 10: Devise a Newton iteration formula for computing $\sqrt[3]{R}$ where $R > 0$. Perform a graphical analysis of your function $f(x)$ to determine the starting values for which the iteration will converge.

§3.2, 15: Consider a variation of Newton's method in which only one derivative is needed; that is,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_0)}$$

Find C and s such that $e_{n+1} = Ce_n^s$.

§3.2, 19*: Prove that if r is a zero of multiplicity k of the function f, then quadratic convergence in Newton's method will be restored by making this modification:

$$x_{n+1} = x_n - k\frac{f(x_n)}{f'(x_n)}$$

§3.2, 23(a): Perform two iterations of Newton's method on this system,

$$4x_1^2 - x_2^2 = 0$$
$$4x_1 x_2^2 - x_1 = 1$$

starting with $(0, 1)$.

§3.3, 7: Prove that the formula for the secant method can be written in the form

$$x_{n+1} = \frac{f(x_n)x_{n-1} - x_n f(x_{n-1})}{f(x_n) - f(x_{n-1})}$$

Explain why this formula is inferior to Equation (3) in practice.