\section*{MATH 170B ASSIGNMENT 2}

\section*{§3.1, 4:} Derive the formula
\[ n \geq \frac{\log(b_0 - a_0) - \log \epsilon}{\log 2} - 1 \]
involving \( b_0 - a_0 \) and \( \epsilon \) for the number of steps \( n \) that must be taken in the bisection method to guarantee that \( |\alpha - \alpha_n| \leq \epsilon \).

\section*{§3.1, 5:} Derive the formula
\[ n \geq \frac{\log(b_0 - a_0) - \log \epsilon - \log a_0}{\log 2} - 1 \]
involving \( a_0, b_0, \) and \( \epsilon \) for the number of steps \( n \) that should be taken in the bisection algorithm to ensure that the root is determined with relative accuracy \( \leq \epsilon \). Assume \( a_0 > 0 \).

\section*{§3.1, 7:} If the bisection method is used starting with the interval \([2, 3]\), how many steps must be taken to compute a root with absolute accuracy \( < 10^{-6} \)? Answer the same question for the relative accuracy. What about to full precision on the \textit{Marc-32} in each case?

\section*{§3.2, 5:} What is the purpose of the following iteration formula?
\[ x_{n+1} = 2x_n - x_n^2 y \]
Identify it as the Newton iteration for a certain function.

\section*{§3.2, 10:} Devise a Newton iteration formula for computing \( \sqrt[3]{R} \) where \( R > 0 \). Perform a graphical analysis of your function \( f(x) \) to determine the starting values for which the iteration will converge.

\section*{§3.2, 15:} Consider a variation of Newton’s method in which only one derivative is needed; that is,
\[ x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \]
Find \( C \) and \( s \) such that \( e_{n+1} = Ce_n^s \).

\section*{§3.2, 19*:} Prove that if \( r \) is a zero of multiplicity \( k \) of the function \( f \), then quadratic convergence in Newton’s method will be restored by making this modification:
\[ x_{n+1} = x_n - k \frac{f(x_n)}{f'(x_n)} \]

\section*{§3.2, 23(a):} Perform two iterations of Newton’s method on this system,
\[ 4x_1^2 - x_2^2 = 0 \]
\[ 4x_1x_2^2 - x_1 = 1 \]
starting with \((0, 1)\).

\section*{§3.3, 7:} Prove that the formula for the secant method can be written in the form
\[ x_{n+1} = \frac{f(x_n)x_{n-1} - x_nf(x_{n-1})}{f(x_n) - f(x_{n-1})} \]
Explain why this formula is inferior to Equation (3) in practice.