(1) Write the theta method,

\[ y_{n+1} = y_n + hf(t_n, y_n) + (1 - \theta) f(t_{n+1}, y_{n+1}) \]

as a Runge–Kutta method, and express it in the form of a Butcher tableau.

(2) Derive the three-stage Runge–Kutta method that corresponds to the collocation points 
\( c_1 = \frac{1}{4}, c_2 = \frac{1}{2}, c_3 = \frac{3}{4} \), and determine its order of accuracy.

(3) Given \( \theta \in [0, 1] \), find the order of the method,

\[ y_{n+1} = y_n + h f(t_n + (1 - \theta) h, \theta y_n + (1 - \theta) y_{n+1}). \]

Remark: Note that this is not the theta method given in the first problem, rather it is a generalization of the midpoint rule, as opposed to the trapezoidal rule.

(4) Provided that \( f \) is analytic, it is possible to obtain from \( y' = f(t, y) \) an expression for the second derivative of \( y \), namely \( y'' = g(t, y) \), where

\[ g(t, y) = \frac{\partial f(t, y)}{\partial t} + f(t, y) \frac{\partial f(t, y)}{\partial y}. \]

Find the orders of the methods

\[ y_{n+1} = y_n + hf(t_n, y_n) + \frac{1}{2} h^2 g(t_n, y_n) \]

and

\[ y_{n+1} = y_n + \frac{1}{2} h f(t_n, y_n) + f(t_{n+1}, y_{n+1}) + \frac{1}{12} h^2 [g(t_n, y_n) - g(t_{n+1}, y_{n+1})]. \]