MATH 170C HOMEWORK 6

(1) Write a MATLAB routine to solve an initial-value problem \( x' = f(t, x) \) with \( x(t_0) = x_0 \) on an interval \( a \leq t \leq b \) using the fourth-order Runge–Kutta method with stepsize \( h \). This function should be written so that it can be called in MATLAB by typing:

\[ [x, t] = \text{RK4}(@f, x0, a, b, h) \]

(a) Consider the following initial-value problem,

\[ x' = \lambda x + \cos t - \lambda \sin t, \quad x(0) = 0 \]

Compare your numerical solution (from RK4) to the exact solution on the interval \([0, 5] \) for different values of \( \lambda = 5, -5, -10 \), and stepsize \( h = 0.01 \). What effect does \( \lambda \) have on the numerical accuracy?

(2) Write a MATLAB routine to solve an initial-value problem \( x' = f(t, x) \) with \( x(t_0) = x_0 \) on an interval \( a \leq t \leq b \) using the fourth-order Adams-Moulton method with stepsize \( h \). This function should be written so that it can be called in MATLAB by typing:

\[ [x, t] = \text{AM4}(@f, x0, a, b, h, \text{TOL, MaxIters}) \]

Use the RK4 method you implemented earlier to obtain the starting values, and use a fixed point iteration to solve the nonlinear equation.

(a) Consider the following initial-value problem,

\[ x' = -2tx^2, \quad x(0) = 1 \]

Compute the solution on the interval \([0, 1] \) with stepsize \( h = 0.25 \) and compare your results with the exact solution \( x(t) = 1/(1 + t^2) \).