1. Consider the following $3 \times 3$ system:

\[
\begin{align*}
4x_1 - x_2 + x_3 &= 7 \\
4x_1 - 8x_2 + x_3 &= -21 \\
-2x_1 + x_2 + 5x_3 &= 15.
\end{align*}
\]

The actual solution to the system is $\vec{x} = (2, 4, 3)^T$.

(a) Write down the Jacobi iteration formula for the above system.

(b) Using the initial guess $\vec{x}^{(0)} = (0, 0, 0)^T$, compute the first four Jacobi iterations for the above system as well as the error $\|\vec{x} - \vec{x}^{(i)}\|_\infty$ for each iterate (NOTE: feel free to use your calculator or MATLAB for this, but make sure to report enough significant digits). Display the data in a table of the form:

<table>
<thead>
<tr>
<th>$i$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1^i$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x_2^i$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x_3^i$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$|\vec{x} - \vec{x}^{(i)}|_\infty$</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

(c) Determine the Gauss-Seidel iterate for the above system.

(d) Repeat part (b) using the Gauss-Seidel iterate from part (c). Display the data in a table of the form:
**NOTE:** \[ \| \vec{x} - \vec{x}^{(i)} \|_{\infty} = \max \left( \| x_1 - x_1^{(i)} \|, \| x_2 - x_2^{(i)} \|, \| x_3 - x_3^{(i)} \| \right) \]

2. **SOURCE CODE:**

Write a MATLAB function for Newton’s method for systems. Write this function so that it works for a 2 × 2 system (in other words: you don’t have to write to solve a general m × m system). Make use of the 2 × 2 inversion formula to invert the necessary linear system in each Newton iteration. Your function should be written so that it can be called in MATLAB by typing:

- \([x, \text{NumIters}] = \text{NewtSys2x2}(@F, @J, x0, \text{TOL}, \text{MaxIters})\)

Here \(F(x_1, x_2)\) is the vector-valued function whose root we are trying to approximate and \(J(x_1, x_2)\) is its Jacobian matrix. Include your source code with the rest of your assignment.

3. **APPLICATION PROBLEM:**

Let \(i = \sqrt{-1}\). Find all five roots of the complex polynomial

\[ f(z) = (1 + i)z^5 - 2z^3 + iz^2 - 1 \]

to within a tolerance of \(10^{-7}\) using your Newton’s method for systems code. For each root report your initial guess, how many iterations were required for a \(10^{-7}\) error, and your approximate root.
HINT: The roots of $f(z)$ are the points $z$ in the complex plane where $f(z) = 0$. We can write $z = x + iy$ where $x$ is the real part and $y$ is the imaginary part. In this form, both $x$ and $y$ are strictly real numbers.

We can also separate $f(z)$ into a real part and an imaginary part:

$$f(z) = (1 + i)(x + iy)^5 - 2(x + iy)^3 + i(x + iy)^2 - 1$$

$$\Rightarrow f(z) = A(x, y) + iB(x, y).$$

where

$$A(x, y) = x^5 - 5x^4y - 10x^3y^2 + 10x^2y^3 + 5xy^4 - y^5 - 2x^3 + 6xy^2 - 2xy - 1$$

$$B(x, y) = x^5 + 5x^4y - 10x^3y^2 - 10x^2y^3 + 5xy^4 + y^5 - 6x^2y + 2y^3 + x^2 - y^2.$$

Again, $x$ and $y$ are strictly real numbers and $A(x, y)$ and $B(x, y)$ are strictly real functions.

The roots of $f(z)$ are all the points $z$ such that $f(z) = 0$. Using the above result this is equivalent to saying that the roots of $f(z)$ are all the points $(x, y)$ such that $A(x, y) = B(x, y) = 0$. 