1. **DISCUSSION QUESTION:**

Suppose that $N(h)$ is an approximation to $M$ for every $h > 0$ and that

$$
M = N(h) + K_1 h^2 + K_2 h^4 + K_3 h^6 + \ldots
$$

for some constants $K_1, K_2, K_3, \ldots$. Use the values $N(h)$, $N(\frac{h}{3})$, and $N(\frac{h}{9})$ to produce an $O(h^6)$ approximation to $M$.

2. **DISCUSSION QUESTION:**

Derive an $O(h^4)$ five-point formula to approximate $f'(x_0)$ that uses $f(x_0 - h)$, $f(x_0)$, $f(x_0 + h)$, $f(x_0 + 2h)$, and $f(x_0 + 3h)$.

3. **SOURCE CODE:**

Write separate MATLAB functions for

- Composite Simpson’s Rule: \( I = \text{Simpson}(@f,a,b,n) \)
- Romberg Integration: \( I = \text{Romberg}(@f,a,b,n) \)

4. **DISCUSSION QUESTION:**

Use your Composite Simpson’s Rule code to approximate the integral

$$
I = \int_0^{10} \cos(2x) \, dx.
$$

Obtain approximations using $n = 2^k$ for $k = 1, 2, 3, \ldots, 10$, and numerical verify that the method has $O(h^4)$ rate of convergence.

For this question, turn in a table that contains 4 columns:

(a) the various $h$ values,
(b) the approximations to $I$,
(c) the errors,
(d) the ratio of the previous/current error.

After you have constructed these tables, write down a brief statement explaining why the results in the table verify that the Composite Simpson’s method is $O(h^4)$. 

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5. **DISCUSSION QUESTION:**

Use your Romberg Integration code to approximate the same integral as in Question #5. Obtain approximations using $n = 4, 5, 6, 7, 8, 9, 10$.

For this question, turn in a table that contains 4 columns:

(a) the various $n$ values,
(b) the approximations to $I$,
(c) the errors,
(d) the ratio of the previous/current error.