Assigned: Friday Sep. 30, 2011
Due Date: Friday Oct. 7, 2011

MATH 174: Homework 2
"Rootfinding"
Fall 2011

NOTE: For each homework assignment observe the following guidelines:

1. Consider the function $f(x) = x^3 - 2$.
   (a) Show that $f(x)$ has a root α in the interval $[1, 2]$.
   (b) Compute an approximation to the root by taking 4 steps of the **bisection method** (**BY HAND**).
   (c) Repeat, using **Newton's method**. Take $x_0 = 1.5$ for the starting value.

   For each method, present the results in the form of a table:
   Column 1: n (step)
   Column 2: x_n (approximation)
   Column 3: $f(x_n)$ (residual)
   Column 4: $|\alpha - x_n|$ (error)

2. **SOURCE CODE:**
   Write separate MATLAB functions for the **bisection method**, the **method of false position**, **Newton's method**, and the **secant method**. These functions should be written so that they can be called in MATLAB by typing:

   - `[x,NumIters] = Bisection(@f,a,b,TOL,MaxIters)`
   - `[x,NumIters] = FalsePos(@f,a,b,TOL,MaxIters)`
   - `[x,NumIters] = Newton(@f,@df,x0,TOL,MaxIters)`
   - `[x,NumIters] = Secant(@f,x0,x1,TOL,MaxIters)`

   Here $f(x)$ is the function whose root we are trying to approximate and $df(x)$ is its derivative. Include your source code with the rest of your assignment.

3. **DISCUSSION QUESTION:**
   Each of the functions

$$f_1(x) = \sin(x) - x - 1$$

$$f_2(x) = x(1 - \cos(x))$$

$$f_3(x) = e^x - x^2 + 3x - 2$$

has a root in the interval $x \in [-2, 1]$. Use all four of the above rootfinding methods to approximate to within an absolute tolerance of 10^{-6} the root of each function. Limit the number of iterations to 500. For Newton's method use the starting value $x_0 = 1$; for the secant method use $x_0 = 1$ and $x_1 = 0.9$. Summarize the results of your analyses in table form:

	Bisection Method		**False Position**	
Function	**Num. Iters.**	**Approx. Root**	**Num. Iters.**	**Approx. Root**
$f_1(x)$				
$f_2(x)$				
$f_3(x)$				

	Newton's Method		**Secant Method**	
Function	**Num. Iters.**	**Approx. Root**	**Num. Iters.**	**Approx. Root**
$f_1(x)$				
$f_2(x)$				
$f_3(x)$				

(a) Why did the bisection method require approximately the same number of iterations to converge to the approximate root for all three test problems?

(b) Newton's method should have experienced difficulty approximating the root of one of the test functions. Identify which function presented a problem and explain why the difficulty occurred.

(c) Above you used the bisection method to find the root of the function $f_1(x) = \sin(x) - x - 1$. Consider the function $g_1(x) = (\sin(x) - x - 1)^2$. Clearly $f_1(x)$ and $g_1(x)$ have the same root in $x \in [-2, 1]$. Could the bisection method be used to numerically approximate the root of $g_1(x)$? Why or why not?

4. **APPLICATION PROBLEM:**

The van der Waal equation

$$\left(P + \frac{a}{V^2}\right)(V - b) = nRT$$

generalizes the ideal gas law $PV = nRT$. In each equation, P represents the pressure (atm), V represents the volume (liters), n is the number of moles of gas, and T represents the temperature (K). R is the universal gas constant and has the value

$$R = 0.08205 \, \frac{\text{liters} \cdot \text{atm}}{\text{mole} \cdot K} \, .$$

Determine the volume of 1 mole of isobutane at a temperature of $T = 313K$ and a pressure of $P = 2$ atm, given that, for isobutane, $a = 12.87$ atm $\cdot$ liters^2 and $b = 0.1142$ liters. Compare this to the value predicted from the ideal gas law. You may use any one of your methods (make clear in your writeup which one you are using, what initial guesses or intervals you are using, etc...).

5. **APPLICATION PROBLEM:**

According to Archimedes' law, when a solid of density σ is placed in a liquid of density ρ, it will sink to a depth h that displaces an amount of liquid whose weight equals the weight of the solid. For a sphere of radius r, Archimedes law translates to

$$\frac{1}{3}\pi\left(3rh^2 - h^3\right)\rho = \frac{4}{3}\pi r^3\sigma.$$

Given $r = 5$, $\rho = 1$, and $\sigma = 0.6$, determine h. You may use any one of your methods (make clear in your writeup which one you are using, what initial guesses or intervals you are using, etc...).